How to find a symmetric object in an ugly minion

Dmitriy Zhuk

Mathematics of Constraint Satisfaction Problems
Durham Symposium 117
April 13-17, 2026

Charles University

European Research Council
Established by the European Commission

Funded by the European Union (ERC, POCOCOP, 101071674)
Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.

Minions

An (abstract) minion $\mathcal{M}$ consists of

- sets $\mathcal{M}^{(1)}, \mathcal{M}^{(2)}, \mathcal{M}^{(3)}, \dots$
- mappings $\mathcal{M}^{(\pi)}\colon \mathcal{M}^{(m)} \to \mathcal{M}^{(n)}$ for all $\pi\colon [m] \to [n]$

such that $\mathcal{M}^{(\mathrm{id}_n)} = \mathrm{id}_{\mathcal{M}^{(n)}}$ and $\mathcal{M}^{(\pi)} \circ \mathcal{M}^{(\pi')} = \mathcal{M}^{(\pi\circ\pi')}$

Minions

An (abstract) minion $\mathcal{M}$ consists of

- sets $\mathcal{M}^{(1)}$, $\mathcal{M}^{(2)}$, $\mathcal{M}^{(3)}, \dots$
- mappings $\mathcal{M}^{(\pi)} \colon \mathcal{M}^{(m)} \to \mathcal{M}^{(n)}$ for all $\pi \colon [m] \to [n]$

such that $\mathcal{M}^{(\mathrm{id}_n)} = \mathrm{id}_{\mathcal{M}^{(n)}}$ and $\mathcal{M}^{(\pi)} \circ \mathcal{M}^{(\pi')} = \mathcal{M}^{(\pi \circ \pi')}$

A clone $\mathcal{C}$

Minions

An (abstract) minion $\mathcal{M}$ consists of

- sets $\mathcal{M}^{(1)}, \mathcal{M}^{(2)}, \mathcal{M}^{(3)}, \dots$
- mappings $\mathcal{M}^{(\pi)} \colon \mathcal{M}^{(m)} \to \mathcal{M}^{(n)}$ for all $\pi \colon [m] \to [n]$

such that $\mathcal{M}^{(\mathrm{id}_n)} = \mathrm{id}_{\mathcal{M}^{(n)}}$ and $\mathcal{M}^{(\pi)} \circ \mathcal{M}^{(\pi')} = \mathcal{M}^{(\pi \circ \pi')}$

A clone $\mathcal{C}$

$\mathcal{C}^{(n)}$:

n-ary operations of $\mathcal{C}$.

Minions

An (abstract) minion $\mathcal{M}$ consists of

- sets $\mathcal{M}^{(1)}, \mathcal{M}^{(2)}, \mathcal{M}^{(3)}, \dots$
- mappings $\mathcal{M}^{(\pi)}\colon \mathcal{M}^{(m)} \to \mathcal{M}^{(n)}$ for all $\pi\colon [m] \to [n]$

such that $\mathcal{M}^{(\mathrm{id}_n)} = \mathrm{id}_{\mathcal{M}^{(n)}}$ and $\mathcal{M}^{(\pi)} \circ \mathcal{M}^{(\pi')} = \mathcal{M}^{(\pi \circ \pi')}$

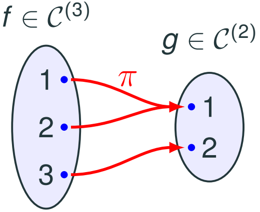
A clone $\mathcal{C}$

$\mathcal{C}^{(n)}$:

n-ary operations of $\mathcal{C}$.

$\mathcal{C}^{(\pi)}(f) = g$, where

$g(x_1, \dots, x_n) = f(x_{\pi(1)}, \dots, x_{\pi(m)})$



$f(x_1, x_1, x_2) = g(x_1, x_2)$

Minions

An *(abstract) minion* $\mathcal{M}$ consists of

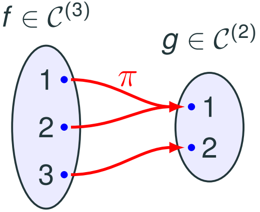
- sets $\mathcal{M}^{(1)}$, $\mathcal{M}^{(2)}$, $\mathcal{M}^{(3)}, \dots$
- mappings $\mathcal{M}^{(\pi)} \colon \mathcal{M}^{(m)} \to \mathcal{M}^{(n)}$ for all $\pi \colon [m] \to [n]$

such that $\mathcal{M}^{(\mathrm{id}_n)} = \mathrm{id}_{\mathcal{M}^{(n)}}$ and $\mathcal{M}^{(\pi)} \circ \mathcal{M}^{(\pi')} = \mathcal{M}^{(\pi \circ \pi')}$

$\mathrm{Pol}(\mathbb{A}, \mathbb{B})$

$\mathrm{Pol}(\mathbb{A}, \mathbb{B})^{(n)}$:
n-ary functions of $\mathrm{Pol}(\mathbb{A}, \mathbb{B})$.

$\mathrm{Pol}(\mathbb{A}, \mathbb{B})^{(\pi)}(f) = g$, where
$g(x_1, \dots, x_n) = f(x_{\pi(1)}, \dots, x_{\pi(m)})$



$f(x_1, x_1, x_2) = g(x_1, x_2)$

Minions

An (abstract) minion $\mathcal{M}$ consists of

- sets $\mathcal{M}^{(1)}, \mathcal{M}^{(2)}, \mathcal{M}^{(3)}, \dots$
- mappings $\mathcal{M}^{(\pi)}\colon \mathcal{M}^{(m)} \to \mathcal{M}^{(n)}$ for all $\pi\colon [m] \to [n]$

such that $\mathcal{M}^{(\mathrm{id}_n)} = \mathrm{id}_{\mathcal{M}^{(n)}}$ and $\mathcal{M}^{(\pi)} \circ \mathcal{M}^{(\pi')} = \mathcal{M}^{(\pi\circ\pi')}$

BLP minion $\mathcal{M}_{\mathrm{BLP}}$

Minions

An *(abstract) minion* $\mathcal{M}$ consists of

- sets $\mathcal{M}^{(1)}, \mathcal{M}^{(2)}, \mathcal{M}^{(3)}, \ldots$
- mappings $\mathcal{M}^{(\pi)} \colon \mathcal{M}^{(m)} \to \mathcal{M}^{(n)}$ for all $\pi \colon [m] \to [n]$

such that $\mathcal{M}^{(\mathrm{id}_n)} = \mathrm{id}_{\mathcal{M}^{(n)}}$ and $\mathcal{M}^{(\pi)} \circ \mathcal{M}^{(\pi')} = \mathcal{M}^{(\pi \circ \pi')}$

BLP minion $\mathcal{M}_{\mathrm{BLP}}$

$\mathcal{M}_{\mathrm{BLP}}^{(n)}$:
vectors $\mathbf{v} \in [0,1]^n$ s.t. $\sum_i \mathbf{v}_i = 1$.

Minions

An *(abstract) minion* $\mathcal{M}$ consists of

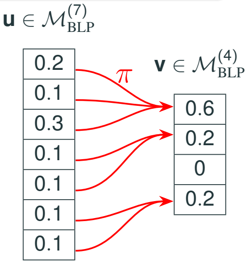
- sets $\mathcal{M}^{(1)}, \mathcal{M}^{(2)}, \mathcal{M}^{(3)}, \dots$
- mappings $\mathcal{M}^{(\pi)} \colon \mathcal{M}^{(m)} \to \mathcal{M}^{(n)}$ for all $\pi \colon [m] \to [n]$

such that $\mathcal{M}^{(\mathrm{id}_n)} = \mathrm{id}_{\mathcal{M}^{(n)}}$ and $\mathcal{M}^{(\pi)} \circ \mathcal{M}^{(\pi')} = \mathcal{M}^{(\pi \circ \pi')}$

BLP minion $\mathcal{M}_{\mathrm{BLP}}$

$\mathcal{M}^{(n)}_{\mathrm{BLP}}$:
vectors $\mathbf{v} \in [0,1]^n$ s.t. $\sum_i \mathbf{v}_i = 1$.

$\mathcal{M}^{(\pi)}_{\mathrm{BLP}}(\mathbf{u}) = \mathbf{v}$, where $\mathbf{v}_i = \sum_{j \in \pi^{-1}(i)} \mathbf{u}_j$



Minions

An *(abstract) minion* $\mathcal{M}$ consists of

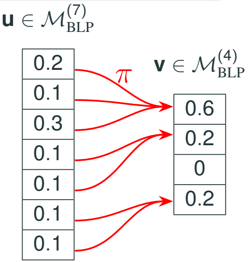
- sets $\mathcal{M}^{(1)}, \mathcal{M}^{(2)}, \mathcal{M}^{(3)}, \dots$
- mappings $\mathcal{M}^{(\pi)}\colon \mathcal{M}^{(m)} \to \mathcal{M}^{(n)}$ for all $\pi\colon [m] \to [n]$

such that $\mathcal{M}^{(\mathrm{id}_n)} = \mathrm{id}_{\mathcal{M}^{(n)}}$ and $\mathcal{M}^{(\pi)} \circ \mathcal{M}^{(\pi')} = \mathcal{M}^{(\pi\circ\pi')}$

BLP minion $\mathcal{M}_{\mathrm{BLP}}$

$\mathcal{M}^{(n)}_{\mathrm{BLP}}$:
vectors $\mathbf{v} \in [0,1]^n$ s.t. $\sum_i \mathbf{v}_i = 1$.

$\mathcal{M}^{(\pi)}_{\mathrm{BLP}}(\mathbf{u}) = \mathbf{v}$, where $\mathbf{v}_i = \sum_{j\in\pi^{-1}(i)} \mathbf{u}_j$

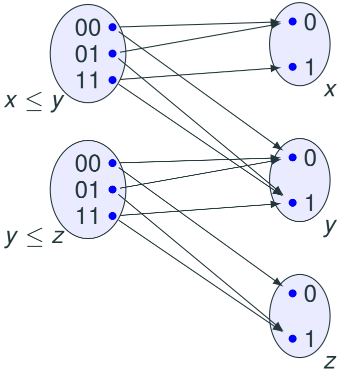


$\xi\colon \mathcal{M} \to \mathcal{N}$ is a homomorphism if $\xi\colon \mathcal{M}^{(n)} \to \mathcal{N}^{(n)}$ for all n and $\xi(\mathcal{M}^{(\pi)}(f)) = \mathcal{N}^{(\pi)}(\xi(f))$ for all $f \in \mathcal{M}^{(m)}$ and $\pi\colon [m] \to [n]$.

Algorithms are minions

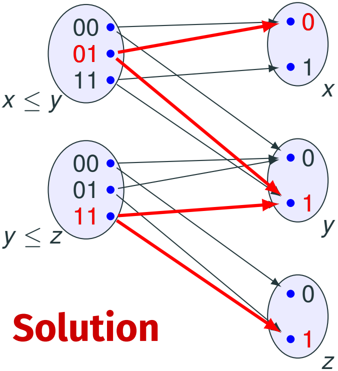
Algorithms are minions

CSP instance $x \le y \wedge y \le z$ on $\{0, 1\}$:



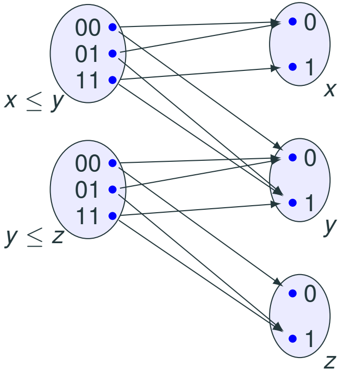
Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:



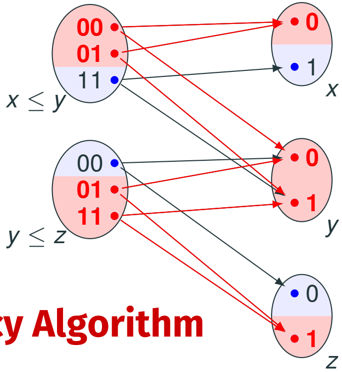
Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:



Algorithms are minions

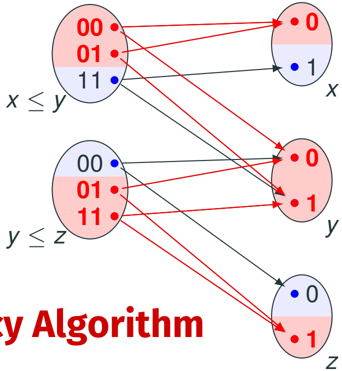
CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:



Arc-consistency Algorithm

Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:



Arc-consistency Algorithm

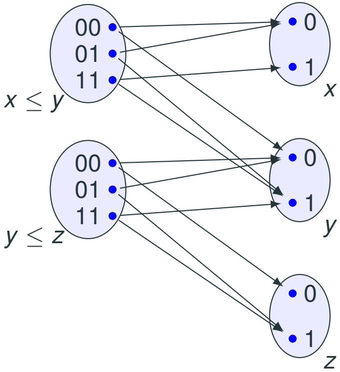
Arc-Consistency algorithm returns “Yes” on an instance IFF the corresponding label cover instance admits $\mathcal{M}_{\text{AC}}$-relaxation.

Arc-Consistency minion: $\mathcal{M}_{\text{AC}}$

n-ary objects are nonempty subsets of $[n]$.

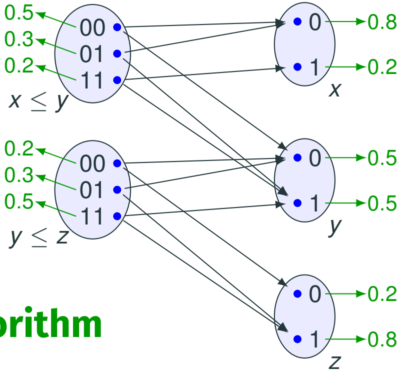
Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:



Algorithms are minions

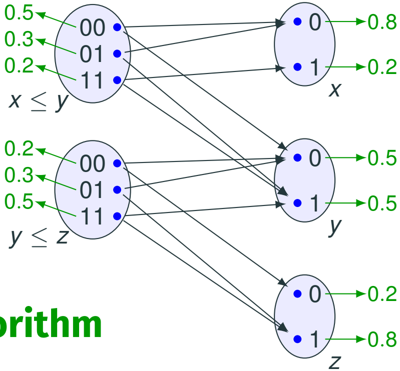
CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:



BLP Algorithm

Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:



BLP Algorithm

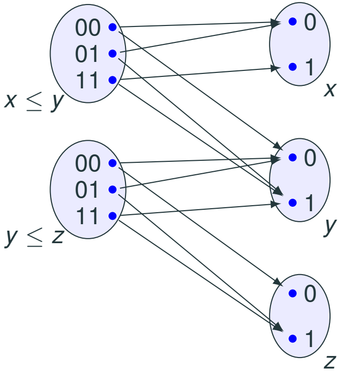
BLP algorithm returns "Yes" on an instance IFF the corresponding label cover instance admits $\mathcal{M}_{\mathrm{BLP}}$-relaxation.

BLP minion $\mathcal{M}_{\mathrm{BLP}}$:

$\mathcal{M}_{\mathrm{BLP}}^{(n)}$ consists of vectors $\mathbf{v} \in [0, 1]^n$ s.t. $\sum_i \mathbf{v}_i = 1$.

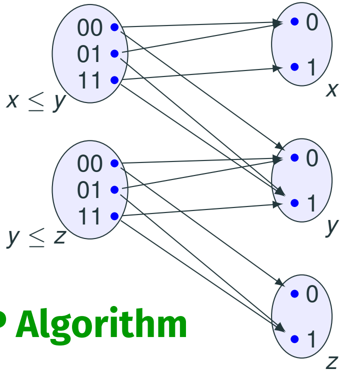
Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:



Algorithms are minions

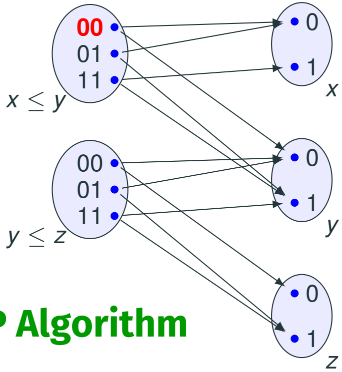
CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:



Singleton BLP Algorithm

Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:

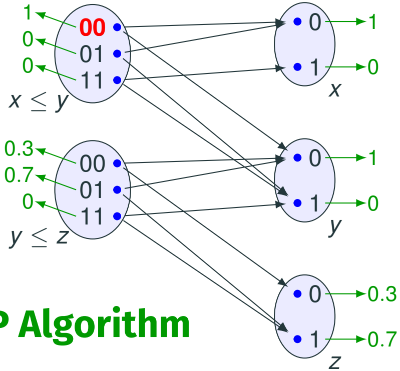


Singleton BLP Algorithm

Singleton BLP algorithm returns “Yes” on an instance IFF the LC instance admits $\mathcal{M}_{\mathrm{BLP}}$-relaxation after fixing any point.

Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:

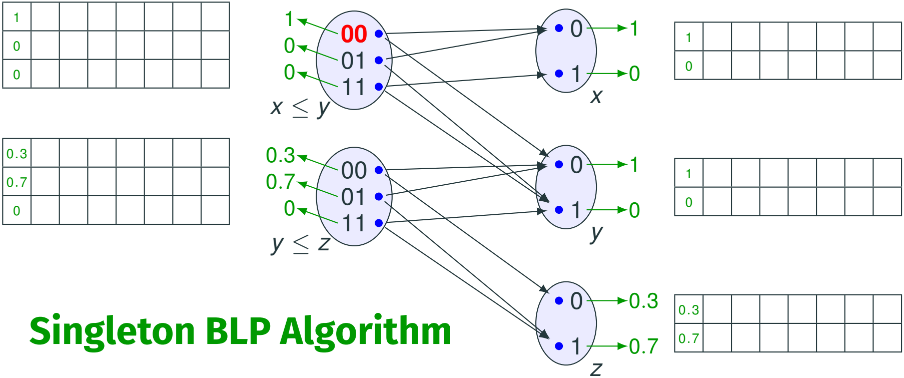


Singleton BLP Algorithm

Singleton BLP algorithm returns "Yes" on an instance IFF the LC instance admits $\mathcal{M}_{\text{BLP}}$-relaxation after fixing any point.

Algorithms are minions

CSP instance $x \le y \wedge y \le z$ on $\{0, 1\}$:



Singleton BLP Algorithm

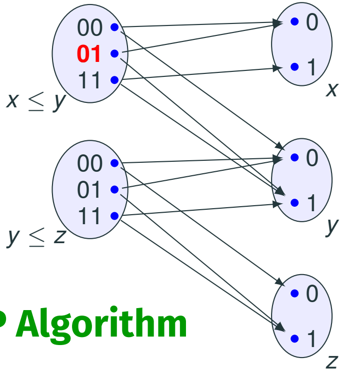
Singleton BLP algorithm returns “Yes” on an instance IFF the LC instance admits $\mathcal{M}_{\mathrm{BLP}}$-relaxation after fixing any point.

Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:

1							
0							
0							

0.3							
0.7							
0							



1							
0							

1							
0							

0.3							
0.7							

Singleton BLP Algorithm

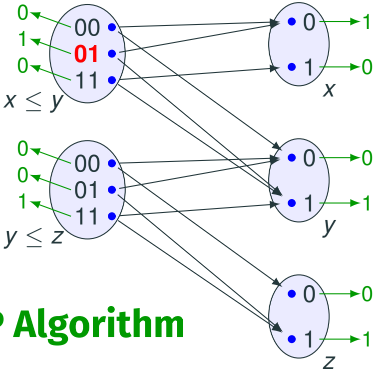
Singleton BLP algorithm returns "Yes" on an instance IFF the LC instance admits $\mathcal{M}_{\text{BLP}}$-relaxation after fixing any point.

Algorithms are minions

CSP instance $x \le y \wedge y \le z$ on $\{0, 1\}$:

1							
0							
0							

0.3							
0.7							
0							



1							
0							

1							
0							

0.3							
0.7							

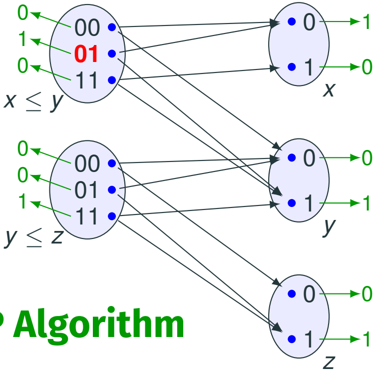
Singleton BLP Algorithm

Singleton BLP algorithm returns "Yes" on an instance IFF the LC instance admits $\mathcal{M}_{\mathrm{BLP}}$-relaxation after fixing any point.

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:

1	0						
0	1						
0	0						

0.3	0						
0.7	0						
0	1						



1	1						
0	0						

1	0						
0	1						

0.3	0						
0.7	1						

Singleton BLP Algorithm

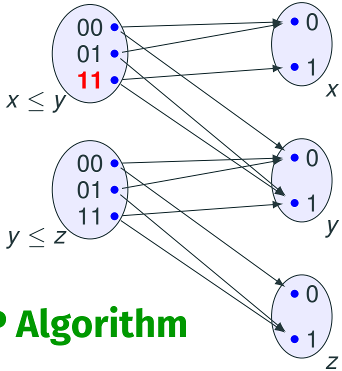
Singleton BLP algorithm returns "Yes" on an instance IFF the LC instance admits $\mathcal{M}_{\mathrm{BLP}}$-relaxation after fixing any point.

Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:

1	0						
0	1						
0	0						

0.3	0						
0.7	0						
0	1						



1	1						
0	0						

1	0						
0	1						

0.3	0						
0.7	1						

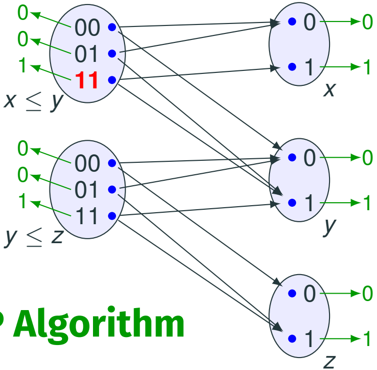
Singleton BLP Algorithm

Singleton BLP algorithm returns “Yes” on an instance IFF the LC instance admits $\mathcal{M}_{\mathrm{BLP}}$-relaxation after fixing any point.

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:

1	0						
0	1						
0	0						

0.3	0						
0.7	0						
0	1						



1	1						
0	0						

1	0						
0	1						

0.3	0						
0.7	1						

Singleton BLP Algorithm

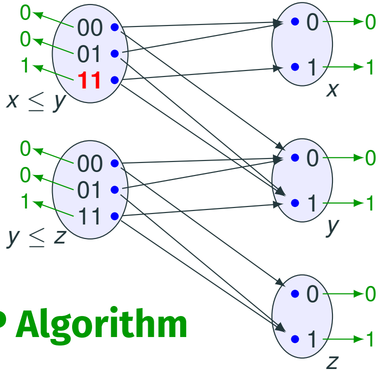
Singleton BLP algorithm returns “Yes” on an instance IFF the LC instance admits $\mathcal{M}_{\mathrm{BLP}}$-relaxation after fixing any point.

Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:

1	0	0					
0	1	0					
0	0	1					

0.3	0	0					
0.7	0	0					
0	1	1					



1	1	0					
0	0	1					

1	0	0					
0	1	1					

0.3	0	0					
0.7	1	1					

Singleton BLP Algorithm

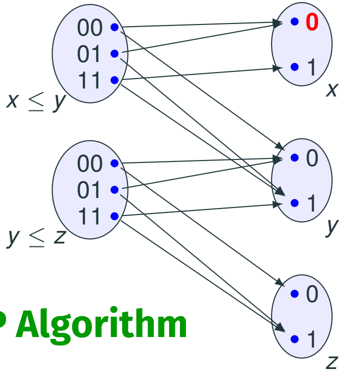
Singleton BLP algorithm returns “Yes” on an instance IFF the LC instance admits $\mathcal{M}_{\mathrm{BLP}}$-relaxation after fixing any point.

Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:

1	0	0					
0	1	0					
0	0	1					

0.3	0	0					
0.7	0	0					
0	1	1					



1	1	0					
0	0	1					

1	0	0					
0	1	1					

0.3	0	0					
0.7	1	1					

Singleton BLP Algorithm

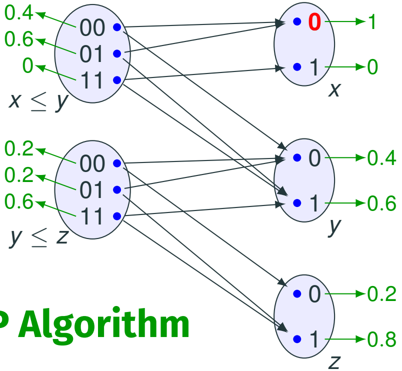
Singleton BLP algorithm returns “Yes” on an instance IFF the LC instance admits $\mathcal{M}_{\mathrm{BLP}}$-relaxation after fixing any point.

Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:

1	0	0					
0	1	0					
0	0	1					

0.3	0	0					
0.7	0	0					
0	1	1					



1	1	0					
0	0	1					

1	0	0					
0	1	1					

0.3	0	0					
0.7	1	1					

Singleton BLP Algorithm

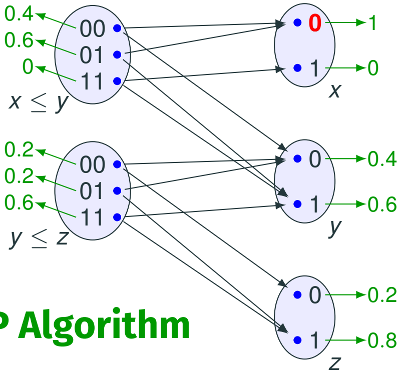
Singleton BLP algorithm returns “Yes” on an instance IFF the LC instance admits $\mathcal{M}_{\mathrm{BLP}}$-relaxation after fixing any point.

Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:

1	0	0	0.4				
0	1	0	0.6				
0	0	1	0				

0.3	0	0	0.2				
0.7	0	0	0.2				
0	1	1	0.6				



1	1	0	1				
0	0	1	0				

1	0	0	0.4				
0	1	1	0.6				

0.3	0	0	0.2				
0.7	1	1	0.8				

Singleton BLP Algorithm

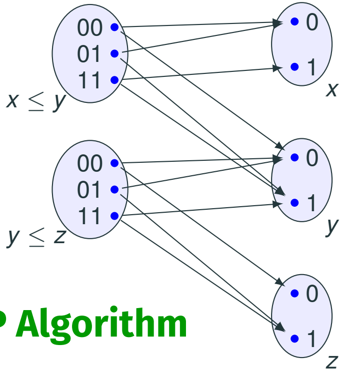
Singleton BLP algorithm returns “Yes” on an instance IFF the LC instance admits $\mathcal{M}_{\mathrm{BLP}}$-relaxation after fixing any point.

Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:

1	0	0	0.4				
0	1	0	0.6				
0	0	1	0				

0.3	0	0	0.2				
0.7	0	0	0.2				
0	1	1	0.6				



1	1	0	1				
0	0	1	0				

1	0	0	0.4				
0	1	1	0.6				

0.3	0	0	0.2				
0.7	1	1	0.8				

Singleton BLP Algorithm

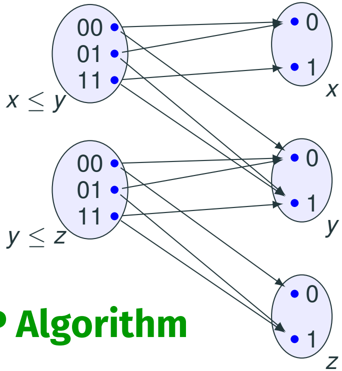
Singleton BLP algorithm returns “Yes” on an instance IFF the LC instance admits $\mathcal{M}_{\mathrm{BLP}}$-relaxation after fixing any point.

Algorithms are minions

CSP instance $x \leq y \wedge y \leq z$ on $\{0, 1\}$:

1	0	0	0.4				
0	1	0	0.6				
0	0	1	0				

0.3	0	0	0.2				
0.7	0	0	0.2				
0	1	1	0.6				



1	1	0	1				
0	0	1	0				

1	0	0	0.4				
0	1	1	0.6				

0.3	0	0	0.2				
0.7	1	1	0.8				

Singleton BLP Algorithm

Singleton BLP algorithm returns "Yes" on an instance IFF the LC instance admits $\mathcal{M}_{\mathrm{BLP}}$-relaxation after fixing any point.

SinglBLP minion $\mathcal{M}_{\mathrm{SinglBLP}}$:

$\mathcal{M}^{(n)}_{\mathrm{SinglBLP}}$ consists of matrices $n \times N$ with entries from $[0, 1]$ s.t. ....

Singleton Minions

SinglBLP minion $\mathcal{M}_{\text{SinglBLP}}$: **[Ciardo, Živný, 2023]**

$\mathcal{M}^{(n)}_{\text{SinglBLP}}$ consists of matrices $n \times N$ with entries from $[0, 1]$ s.t.

Singleton Minions

SinglBLP minion $\mathcal{M}_{\text{SinglBLP}}$: **[Ciardo, Živný, 2023]**

$\mathcal{M}^{(n)}_{\text{SinglBLP}}$ consists of matrices $n \times N$ with entries from $[0, 1]$ s.t.

- Columns sum to 1.

Singleton Minions

SinglBLP minion $\mathcal{M}_{\text{SinglBLP}}$: **[Ciardo, Živný, 2023]**

$\mathcal{M}^{(n)}_{\text{SinglBLP}}$ consists of matrices $n \times N$ with entries from $[0, 1]$ s.t.

- Columns sum to 1.
- Each non-zero row contains 1 (Skeletal).

Singleton Minions

SinglBLP minion $\mathcal{M}_{\text{SinglBLP}}$: **[Ciardo, Živný, 2023]**

$\mathcal{M}^{(n)}_{\text{SinglBLP}}$ consists of matrices $n \times N$ with entries from $[0, 1]$ s.t.

- Columns sum to 1.
- Each non-zero row contains 1 (Skeletal).

0.5	**1**	0.2	0.3	**0**	0.1	0.6	1	0.8	0.9	0.3	**0**	0.5	0.4	0.6
0.5	**0**	0.3	0.3	**0**	0.4	0.2	0	0.1	0.1	0.5	**1**	0.3	0.2	0.2
0	**0**	0	0	**0**	0	0	0	0	0	0	**0**	0	0	0
0	**0**	0.5	0.4	**1**	0.5	0.2	0	0.1	0	0.2	**0**	0.2	0.4	0.2

Singleton Minions

SinglBLP minion $\mathcal{M}_{\text{SinglBLP}}$: [Ciardo, Živný, 2023]

$\mathcal{M}^{(n)}_{\text{SinglBLP}}$ consists of matrices $n \times N$ with entries from $[0, 1]$ s.t.

- Columns sum to 1.
- Each non-zero row contains 1 (Skeletal).

0.5	**1**	0.2	0.3	**0**	0.1	0.6	1	0.8	0.9	0.3	**0**	0.5	0.4	0.6
0.5	**0**	0.3	0.3	**0**	0.4	0.2	0	0.1	0.1	0.5	**1**	0.3	0.2	0.2
0	**0**	0	0	**0**	0	0	0	0	0	0	**0**	0	0	0
0	**0**	0.5	0.4	**1**	0.5	0.2	0	0.1	0	0.2	**0**	0.2	0.4	0.2

SinglAC minion $\mathcal{M}_{\text{SinglAC}}$: [Ciardo, Živný, 2023]

$\mathcal{M}^{(n)}_{\text{SinglAC}}$ consists of matrices $n \times N$ with entries in $\{0, 1\}$ s.t.

Singleton Minions

SinglBLP minion $\mathcal{M}_{\text{SinglBLP}}$: [Ciardo, Živný, 2023]

$\mathcal{M}^{(n)}_{\text{SinglBLP}}$ consists of matrices $n \times N$ with entries from $[0, 1]$ s.t.

- Columns sum to 1.
- Each non-zero row contains 1 (Skeletal).

0.5	**1**	0.2	0.3	**0**	0.1	0.6	1	0.8	0.9	0.3	**0**	0.5	0.4	0.6
0.5	**0**	0.3	0.3	**0**	0.4	0.2	0	0.1	0.1	0.5	**1**	0.3	0.2	0.2
0	**0**	0	0	**0**	0	0	0	0	0	0	**0**	0	0	0
0	**0**	0.5	0.4	**1**	0.5	0.2	0	0.1	0	0.2	**0**	0.2	0.4	0.2

SinglAC minion $\mathcal{M}_{\text{SinglAC}}$: [Ciardo, Živný, 2023]

$\mathcal{M}^{(n)}_{\text{SinglAC}}$ consists of matrices $n \times N$ with entries in $\{0, 1\}$ s.t.

- Each column has 1.

Singleton Minions

SinglBLP minion $\mathcal{M}_{\text{SinglBLP}}$: [Ciardo, Živný, 2023]

$\mathcal{M}^{(n)}_{\text{SinglBLP}}$ consists of matrices $n \times N$ with entries from $[0, 1]$ s.t.

- Columns sum to 1.
- Each non-zero row contains 1 (Skeletal).

0.5	**1**	0.2	0.3	**0**	0.1	0.6	1	0.8	0.9	0.3	**0**	0.5	0.4	0.6
0.5	**0**	0.3	0.3	**0**	0.4	0.2	0	0.1	0.1	0.5	**1**	0.3	0.2	0.2
0	**0**	0	0	**0**	0	0	0	0	0	0	**0**	0	0	0
0	**0**	0.5	0.4	**1**	0.5	0.2	0	0.1	0	0.2	**0**	0.2	0.4	0.2

SinglAC minion $\mathcal{M}_{\text{SinglAC}}$: [Ciardo, Živný, 2023]

$\mathcal{M}^{(n)}_{\text{SinglAC}}$ consists of matrices $n \times N$ with entries in $\{0, 1\}$ s.t.

- Each column has 1.
- Each nonzero row has a column with zeros elsewhere (Skeletal).

Singleton Minions

SinglBLP minion $\mathcal{M}_{\text{SinglBLP}}$: [Ciardo, Živný, 2023]

$\mathcal{M}^{(n)}_{\text{SinglBLP}}$ consists of matrices $n \times N$ with entries from $[0, 1]$ s.t.

- Columns sum to 1.
- Each non-zero row contains 1 (Skeletal).

0.5	**1**	0.2	0.3	**0**	0.1	0.6	1	0.8	0.9	0.3	**0**	0.5	0.4	0.6
0.5	**0**	0.3	0.3	**0**	0.4	0.2	0	0.1	0.1	0.5	**1**	0.3	0.2	0.2
0	**0**	0	0	**0**	0	0	0	0	0	0	**0**	0	0	0
0	**0**	0.5	0.4	**1**	0.5	0.2	0	0.1	0	0.2	**0**	0.2	0.4	0.2

SinglAC minion $\mathcal{M}_{\text{SinglAC}}$: [Ciardo, Živný, 2023]

$\mathcal{M}^{(n)}_{\text{SinglAC}}$ consists of matrices $n \times N$ with entries in $\{0, 1\}$ s.t.

- Each column has 1.
- Each nonzero row has a column with zeros elsewhere (Skeletal).

1	0	**1**	0	0	1	0	**0**	0	0	1	1	0	0	**0**	0	1	0	1	0	**0**	0	0	1
0	1	**0**	0	0	1	0	**1**	0	1	0	0	1	0	**0**	0	0	1	0	0	**0**	1	0	0
0	0	**0**	1	0	0	1	**0**	0	1	0	0	0	0	**1**	1	0	0	1	0	**0**	0	1	0
0	0	**0**	0	1	0	0	**0**	1	0	0	1	0	1	**0**	1	0	0	0	1	**1**	0	0	0

Are minions useful?

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then
$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

- $\mathcal{M}_{\mathrm{BLP}}^{(n)}$ contains $(\frac{1}{n}, \ldots, \frac{1}{n})$.

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

- $\mathcal{M}_{\mathrm{BLP}}^{(n)}$ contains $(\frac{1}{n}, \dots, \frac{1}{n})$.
- $\mathcal{M}_{\mathrm{BLP}} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B}) \ \Rightarrow \ \mathrm{Pol}(\mathbb{A}, \mathbb{B})$ contains an n-ary symmetric function for every $n \in \mathbb{N}$.

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

- $\mathcal{M}_{\mathrm{BLP}}^{(n)}$ contains $(\frac{1}{n}, \dots, \frac{1}{n})$.
- $\mathcal{M}_{\mathrm{BLP}} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B}) \;\Rightarrow\; \mathrm{Pol}(\mathbb{A}, \mathbb{B})$ contains an n-ary symmetric function for every $n \in \mathbb{N}$.
- symmetric functions of all arities $\Rightarrow$ BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

- $\mathcal{M}_{\mathrm{BLP}}^{(n)}$ contains $(\frac{1}{n}, \dots, \frac{1}{n})$.
- $\mathcal{M}_{\mathrm{BLP}} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B}) \Rightarrow \mathrm{Pol}(\mathbb{A}, \mathbb{B})$ contains an n-ary symmetric function for every $n \in \mathbb{N}$.
- symmetric functions of all arities $\Rightarrow$ BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.

Theorem [BBKO, 2021]

BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$ IFF $\mathrm{Pol}(\mathbb{A}, \mathbb{B})$ contains symmetric functions of all arities.

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

How to use $\mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$?

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A},\mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$$

How to use $\mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$?

1	0	**1**	0	0	1	0	**0**	0	0	1	1	0	0	**0**	0	1	0	1	0	**0**	0	0	1
0	1	**0**	0	0	1	0	**1**	0	1	0	0	1	0	**0**	0	0	1	0	0	**0**	1	0	0
0	0	**0**	1	0	0	1	**0**	0	1	0	0	0	0	**1**	1	0	0	1	0	**0**	0	1	0
0	0	**0**	0	1	0	0	**0**	1	0	0	1	0	1	**0**	1	0	0	0	1	**1**	0	0	0

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A},\mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$$

How to use $\mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$?

1	0	**1**	0	0	1	0	**0**	0	0	1	1	0	0	**0**	0	1	0	1	0	**0**	0	0	1
0	1	**0**	0	0	1	0	**1**	0	1	0	0	1	0	**0**	0	0	1	0	0	**0**	1	0	0
0	0	**0**	1	0	0	1	**0**	0	1	0	0	0	0	**1**	1	0	0	1	0	**0**	0	1	0
0	0	**0**	0	1	0	0	**0**	1	0	0	1	0	1	**0**	1	0	0	0	1	**1**	0	0	0

- Any identity satisfied in $\mathcal{M}_{\mathrm{SinglAC}}$ and having just one functional symbol is trivial.

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

How to use $\mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$?

1	0	**1**	0	0	1	0	**0**	0	0	1	1	0	0	**0**	0	1	0	1	0	**0**	0	0	1
0	1	**0**	0	0	1	0	**1**	0	1	0	0	1	0	**0**	0	0	1	0	0	**0**	1	0	0
0	0	**0**	1	0	0	1	**0**	0	1	0	0	0	0	**1**	1	0	0	1	0	**0**	0	1	0
0	0	**0**	0	1	0	0	**0**	1	0	0	1	0	1	**0**	1	0	0	0	1	**1**	0	0	0

- Any identity satisfied in $\mathcal{M}_{\mathrm{SinglAC}}$ and having just one functional symbol is trivial.

Idea

If $\mathrm{Pol}(\mathbb{A}, \mathbb{B})$ is locally finite, many objects must collapse when we map $\mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$.

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

Suppose $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

Suppose $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$

$$\text{Let } \Sigma = \left\{ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \dots, \right\}$$

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

Suppose $\xi \colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$

Let $\Sigma = \left\{ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \dots, \right\}$

$f \colon \Sigma^N \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})^{(4)}$,

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A},\mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$$

Suppose $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$

Let $\Sigma = \left\{ \begin{bmatrix} 1&0&1&0\\ 0&1&0&0\\ 0&0&0&1\\ 0&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&1&0&0\\ 0&1&0&1\\ 0&0&1&0\\ 1&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&1&1\\ 0&1&0&0\\ 0&1&0&0\\ 1&0&0&1 \end{bmatrix}, \begin{bmatrix} 0&0&0&0\\ 1&0&0&0\\ 0&0&1&1\\ 0&1&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&1&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix}, \begin{bmatrix} 0&0&0&1\\ 0&1&0&0\\ 0&0&1&0\\ 1&0&0&0 \end{bmatrix}, \dots, \right\}$

$f\colon \Sigma^N \to \mathrm{Pol}(\mathbb{A},\mathbb{B})^{(4)}$, $f(M_1,\dots,M_N) = \xi(M_1|\dots|M_N)$

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A},\mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$$

Suppose $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$

Let $\Sigma = \left\{ \begin{bmatrix} 1&0&1&0\\0&1&0&0\\0&0&0&1\\0&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&1&0&0\\0&1&0&1\\0&0&1&0\\1&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&1&1\\0&1&0&0\\0&1&0&0\\1&0&0&1 \end{bmatrix}, \begin{bmatrix} 0&0&0&0\\1&0&0&0\\0&0&1&1\\0&1&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&1&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{bmatrix}, \begin{bmatrix} 0&0&0&1\\0&1&0&0\\0&0&1&0\\1&0&0&0 \end{bmatrix}, \dots, \right\}$

$f\colon \Sigma^N \to \mathrm{Pol}(\mathbb{A},\mathbb{B})^{(4)}$, $f(M_1,\dots,M_N) = \xi(M_1|\dots|M_N)$

ξ :

1	0	1	0	0	1	0	0	0	0	1	1	0	1	0	0	1	0	1	0	0	0	0	1
0	1	0	0	0	1	0	1	0	1	0	0	0	1	0	1	0	1	0	0	0	1	0	0
0	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0

$\to \mathrm{Pol}(\mathbb{A},\mathbb{B})$

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A},\mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$$

Suppose $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$

$$\text{Let } \Sigma = \left\{ \begin{bmatrix}1&0&1&0\\0&1&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}, \begin{bmatrix}0&1&0&0\\0&1&0&1\\0&0&1&0\\1&0&0&0\end{bmatrix}, \begin{bmatrix}0&0&1&1\\0&1&0&0\\0&1&0&0\\1&0&0&1\end{bmatrix}, \begin{bmatrix}0&0&0&0\\1&0&0&0\\0&0&1&1\\0&1&0&1\end{bmatrix}, \begin{bmatrix}1&0&1&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}, \begin{bmatrix}0&0&0&1\\0&1&0&0\\0&0&1&0\\1&0&0&0\end{bmatrix}, \dots, \right\}$$

$f\colon \Sigma^N \to \mathrm{Pol}(\mathbb{A},\mathbb{B})^{(4)}$, $f(M_1,\dots,M_N) = \xi(M_1|\dots|M_N)$

By the Hales-Jewett Theorem, if N is large enough,
$f(M_1, X, M_3, X, \dots)$ does not depend on X.

ξ :

1	0	1	0	0	1	0	0	0	0	1	1	0	1	0	0	1	0	1	0	0	0	0	1
0	1	0	0	0	1	0	1	0	1	0	0	0	1	0	1	0	1	0	0	0	1	0	0
0	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0

$\to \mathrm{Pol}(\mathbb{A},\mathbb{B})$

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

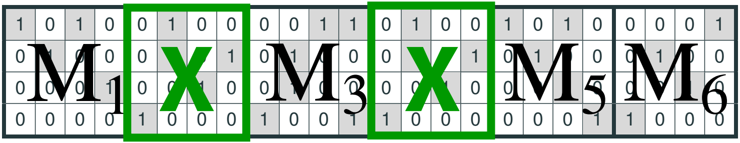
$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A},\mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$$

Suppose $\xi \colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$

Let $\Sigma = \left\{ \begin{bmatrix} 1&0&1&0\\0&1&0&0\\0&0&0&1\\0&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&1&0&0\\0&1&0&1\\0&0&1&0\\1&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&1&1\\0&1&0&0\\0&1&0&0\\1&0&0&1 \end{bmatrix}, \begin{bmatrix} 0&0&0&0\\1&0&0&0\\0&0&1&1\\0&1&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&1&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{bmatrix}, \begin{bmatrix} 0&0&0&1\\0&1&0&0\\0&0&1&0\\1&0&0&0 \end{bmatrix}, \dots, \right\}$

$f \colon \Sigma^N \to \mathrm{Pol}(\mathbb{A},\mathbb{B})^{(4)}$, $f(M_1,\dots,M_N) = \xi(M_1|\dots|M_N)$

By the Hales-Jewett Theorem, if N is large enough,
$f(M_1, X, M_3, X, \dots)$ does not depend on X.

ξ : 

$\to g$

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A},\mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$$

Suppose $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$

Let $\Sigma = \left\{ \begin{bmatrix} 1&0&1&0\\0&1&0&0\\0&0&0&1\\0&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&1&0&0\\0&1&0&1\\0&0&1&0\\1&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&1&1\\0&1&0&0\\0&1&0&0\\1&0&0&1 \end{bmatrix}, \begin{bmatrix} 0&0&0&0\\1&0&0&0\\0&0&1&1\\0&1&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&1&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{bmatrix}, \begin{bmatrix} 0&0&0&1\\0&1&0&0\\0&0&1&0\\1&0&0&0 \end{bmatrix}, \dots, \right\}$

$f\colon \Sigma^N \to \mathrm{Pol}(\mathbb{A},\mathbb{B})^{(4)}$, $f(M_1,\dots,M_N) = \xi(M_1|\dots|M_N)$

By the Hales-Jewett Theorem, if N is large enough,
$f(M_1, X, M_3, X, \dots)$ does not depend on X.

ξ:

1	0	1	0	0	1	0	0	0	0	1	1	0	1	0	0	1	0	1	0	0	0	0	1
0	1	0	0	0	1	0	1	0	1	0	0	0	1	0	1	0	1	0	0	0	1	0	0
0	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0

$\to g$

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

Suppose $\xi \colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$

$$\text{Let } \Sigma = \left\{ \begin{bmatrix} 1&0&1&0\\0&1&0&0\\0&0&0&1\\0&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&1&0&0\\0&1&0&1\\0&0&1&0\\1&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&1&1\\0&1&0&0\\0&1&0&0\\1&0&0&1 \end{bmatrix}, \begin{bmatrix} 0&0&0&0\\1&0&0&0\\0&0&1&1\\0&1&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&1&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{bmatrix}, \begin{bmatrix} 0&0&0&1\\0&1&0&0\\0&0&1&0\\1&0&0&0 \end{bmatrix}, \dots, \right\}$$

$f \colon \Sigma^N \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})^{(4)}$, $f(M_1, \dots, M_N) = \xi(M_1 | \dots | M_N)$

By the Hales-Jewett Theorem, if N is large enough,
$f(M_1, X, M_3, X, \dots)$ does not depend on X.

$$\xi \colon \left[\begin{array}{cccc|cccc|cccc|cccc|cccc|cccc} 1&0&1&0&1&0&0&0&0&0&1&1&1&0&0&0&1&0&1&0&0&0&0&1\\ 0&1&0&0&0&0&1&1&0&1&0&0&0&0&1&1&0&1&0&0&0&1&0&0\\ 0&0&0&1&0&1&0&0&0&1&0&0&0&1&0&0&0&0&1&0&0&0&1&0\\ 0&0&0&0&0&0&0&1&1&0&0&1&0&0&0&1&0&0&0&1&1&0&0&0 \end{array}\right] \to g$$

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A}, \mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$$

Suppose $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})$

Let $\Sigma = \left\{ \begin{bmatrix} 1&0&1&0\\0&1&0&0\\0&0&0&1\\0&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&1&0&0\\0&1&0&1\\0&0&1&0\\1&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&1&1\\0&1&0&0\\0&1&0&0\\1&0&0&1 \end{bmatrix}, \begin{bmatrix} 0&0&0&0\\1&0&0&0\\0&0&1&1\\0&1&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&1&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{bmatrix}, \begin{bmatrix} 0&0&0&1\\0&1&0&0\\0&0&1&0\\1&0&0&0 \end{bmatrix}, \dots, \right\}$

$f\colon \Sigma^N \to \mathrm{Pol}(\mathbb{A}, \mathbb{B})^{(4)}$, $f(M_1, \dots, M_N) = \xi(M_1 | \dots | M_N)$

By the Hales-Jewett Theorem, if N is large enough,
$f(M_1, X, M_3, X, \dots)$ does not depend on X.

ξ :

1	0	1	0	0	0	1	0	0	0	1	1	0	0	1	0	1	0	1	0	0	0	0	1
0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	1	0	0
0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	1	0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0

$\to g$

Are minions useful?

Observation

Suppose an algorithm $\mathfrak{A}$ is equivalent to $\mathcal{M}$-relaxation. Then

$$\mathfrak{A} \text{ solves } \mathrm{PCSP}(\mathbb{A},\mathbb{B}) \iff \mathcal{M} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$$

Suppose $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathrm{Pol}(\mathbb{A},\mathbb{B})$

$$\text{Let } \Sigma = \left\{ \begin{bmatrix}1&0&1&0\\0&1&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}, \begin{bmatrix}0&1&0&0\\0&1&0&1\\0&0&1&0\\1&0&0&0\end{bmatrix}, \begin{bmatrix}0&0&1&1\\0&1&0&0\\0&1&0&0\\1&0&0&1\end{bmatrix}, \begin{bmatrix}0&0&0&0\\1&0&0&0\\0&0&1&1\\0&1&0&1\end{bmatrix}, \begin{bmatrix}1&0&1&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}, \begin{bmatrix}0&0&0&1\\0&1&0&0\\0&0&1&0\\1&0&0&0\end{bmatrix}, \dots, \right\}$$

$f\colon \Sigma^N \to \mathrm{Pol}(\mathbb{A},\mathbb{B})^{(4)}$, $f(M_1,\dots,M_N) = \xi(M_1|\dots|M_N)$

By the Hales-Jewett Theorem, if N is large enough,
$f(M_1, X, M_3, X, \dots)$ does not depend on X.

ξ:

1	0	1	0	1	0	0	0	0	0	1	1	1	0	0	0	1	0	1	0	0	0	0	1
0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0
0	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0

$\to g$

Minion homomorphism $\xi\colon \mathcal{M}_{\text{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.

Minion homomorphism $\xi\colon \mathcal{M}_{\text{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.

0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0

$\notin \mathcal{M}_{\text{SinglAC}}$

Minion homomorphism $\xi\colon \mathcal{M}_{\text{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.

0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0

$\notin \mathcal{M}_{\text{SinglAC}}$

π_1

1	1	0	1	1	0	1	0	1	1	0	1	0	1	1	0	1	1	1	0	1	1	0	1
1	0	1	0	1	1	0	1	1	0	1	1	1	0	1	1	0	1	0	1	1	0	1	1

Minor maps: π_1

Minion homomorphism $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.

π_1

0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0

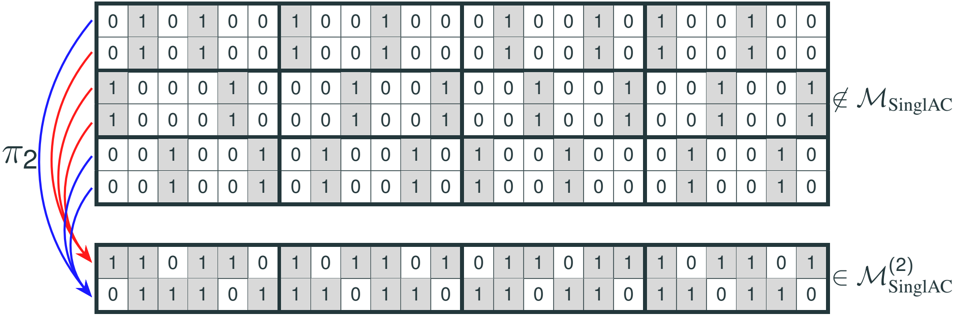
$\notin \mathcal{M}_{\mathrm{SinglAC}}$

1	1	0	1	1	0	1	0	1	1	0	1	0	1	1	0	1	1	1	0	1	1	0	1
1	0	1	0	1	1	0	1	1	0	1	1	1	0	1	1	0	1	0	1	1	0	1	1

$\in \mathcal{M}^{(2)}_{\mathrm{SinglAC}}$

Minor maps: π_1

Minion homomorphism $\xi\colon \mathcal{M}_{\text{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.



Minor maps: π_1, π_2,

Minion homomorphism $\xi\colon \mathcal{M}_{\text{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.

π_3

0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0

$\notin \mathcal{M}_{\text{SinglAC}}$

0	1	1	1	0	1	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0
1	1	0	1	1	0	1	0	1	1	0	1	0	1	1	0	1	1	1	0	1	1	0	1

$\in \mathcal{M}^{(2)}_{\text{SinglAC}}$

Minor maps: π_1, π_2, π_3,

Minion homomorphism $\xi\colon \mathcal{M}_{\text{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.

π_3

0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0

$\notin \mathcal{M}_{\text{SinglAC}}$

0	1	1	1	0	1	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0
1	1	0	1	1	0	1	0	1	1	0	1	0	1	1	0	1	1	1	0	1	1	0	1

$\in \mathcal{M}^{(2)}_{\text{SinglAC}}$

Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

Minion homomorphism $\xi\colon \mathcal{M}_{\text{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.

0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0

$\notin \mathcal{M}_{\text{SinglAC}}$

π_1

1	1	0	1	1	0	1	0	1	1	0	1	0	1	1	0	1	1	1	0	1	1	0	1
1	0	1	0	1	1	0	1	1	0	1	1	1	0	1	1	0	1	0	1	1	0	1	1

$\in \mathcal{M}^{(2)}_{\text{SinglAC}}$

Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

Minion homomorphism $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.

π_1

0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0

$\notin \mathcal{M}_{\mathrm{SinglAC}}$

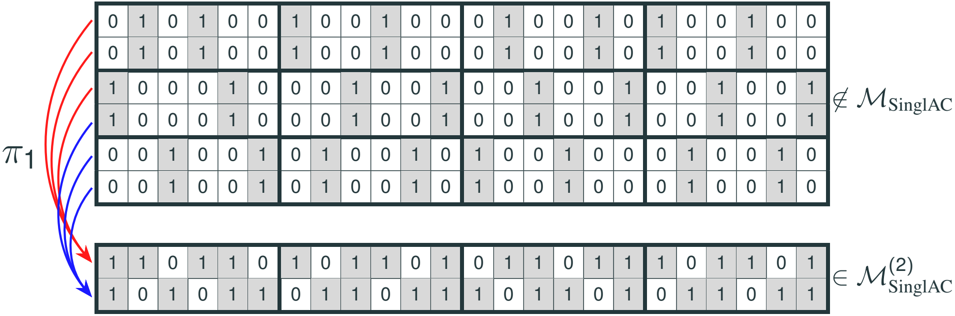
1	1	0	1	1	0	1	0	1	1	0	1	0	1	1	0	1	1	1	0	1	1	0	1
1	0	1	0	1	1	0	1	1	0	1	1	1	0	1	1	0	1	0	1	1	0	1	1

$\in \mathcal{M}^{(2)}_{\mathrm{SinglAC}}$

Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

$$\Sigma = \left\{ \begin{pmatrix} 0&1&0&1&0&0\\ 0&1&0&1&0&0\\ 1&0&0&0&1&0\\ 1&0&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1 \end{pmatrix}, \begin{pmatrix} 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0 \end{pmatrix}, \begin{pmatrix} 0&1&0&0&1&0\\ 0&1&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 1&0&0&1&0&0\\ 1&0&0&1&0&0 \end{pmatrix}, \begin{pmatrix} 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0 \end{pmatrix}, \ldots \right\},$$

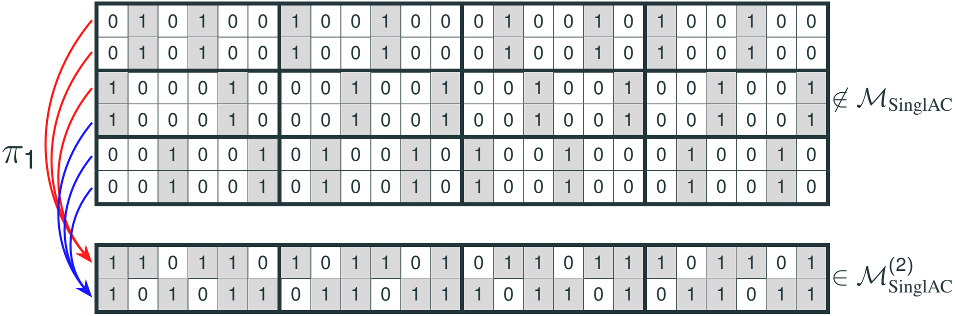
Minion homomorphism $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.



Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

$$\Sigma = \left\{ \begin{smallmatrix} 0&1&0&1&0&0\\0&1&0&1&0&0\\1&0&0&0&1&0\\1&0&0&0&1&0\\0&0&1&0&0&1\\0&0&1&0&0&1 \end{smallmatrix}, \begin{smallmatrix} 1&0&0&1&0&0\\1&0&0&1&0&0\\0&0&1&0&0&1\\0&0&1&0&0&1\\0&1&0&0&1&0\\0&1&0&0&1&0 \end{smallmatrix}, \begin{smallmatrix} 0&1&0&0&1&0\\0&1&0&0&1&0\\0&0&1&0&0&1\\0&0&1&0&0&1\\1&0&0&1&0&0\\1&0&0&1&0&0 \end{smallmatrix}, \begin{smallmatrix} 1&0&0&1&0&0\\1&0&0&1&0&0\\0&0&1&0&0&1\\0&0&1&0&0&1\\0&1&0&0&1&0\\0&1&0&0&1&0 \end{smallmatrix}, \ldots \right\}, C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}.$$

Minion homomorphism $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.

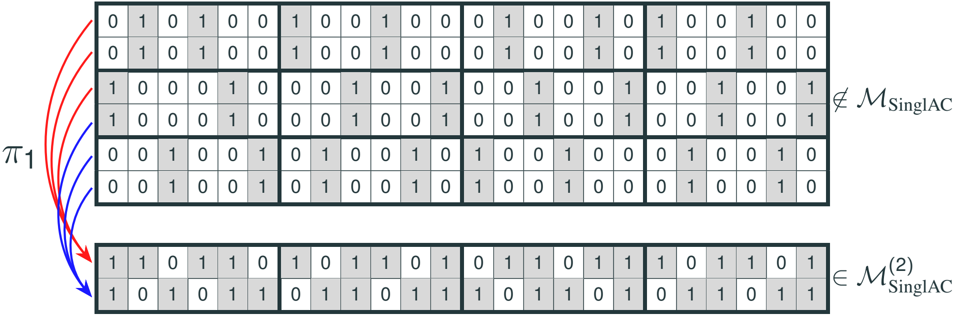


Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

$$\Sigma = \left\{ \begin{bmatrix} 0&1&0&1&0&0\\ 0&1&0&1&0&0\\ 1&0&0&0&1&0\\ 1&0&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0 \end{bmatrix}, \begin{bmatrix} 0&1&0&0&1&0\\ 0&1&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 1&0&0&1&0&0\\ 1&0&0&1&0&0 \end{bmatrix}, \begin{bmatrix} 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0 \end{bmatrix}, \ldots \right\}, C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}.$$

$f\colon \Sigma^N \to C,$

Minion homomorphism $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.

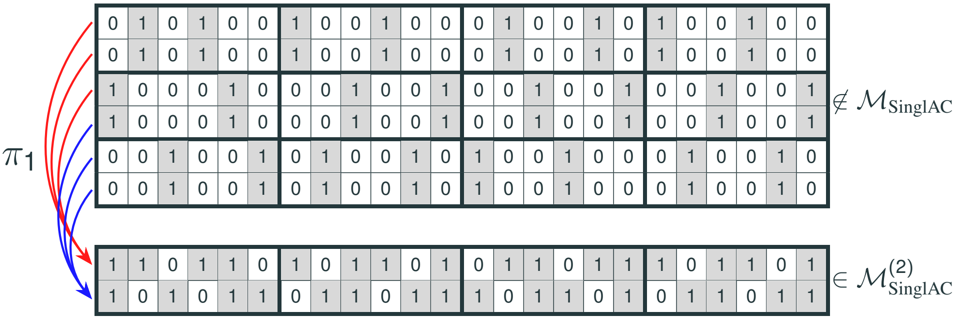


Minor maps: $\pi_1, \pi_2, \pi_3, \dots, \pi_{12}$.

$$\Sigma = \left\{ \begin{pmatrix} 0&1&0&1&0&0\\ 0&1&0&1&0&0\\ 1&0&0&0&1&0\\ 1&0&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1 \end{pmatrix}, \begin{pmatrix} 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0 \end{pmatrix}, \begin{pmatrix} 0&1&0&0&1&0\\ 0&1&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 1&0&0&1&0&0\\ 1&0&0&1&0&0 \end{pmatrix}, \begin{pmatrix} 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0 \end{pmatrix}, \dots \right\}, \; C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}.$$

$$f\colon \Sigma^N \to C,\; f(M_1, \dots, M_N) = (\xi(M_1|\dots|M_N)^{\pi_1}, \dots, \xi(M_1|\dots|M_N)^{\pi_{12}})$$

Minion homomorphism $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.



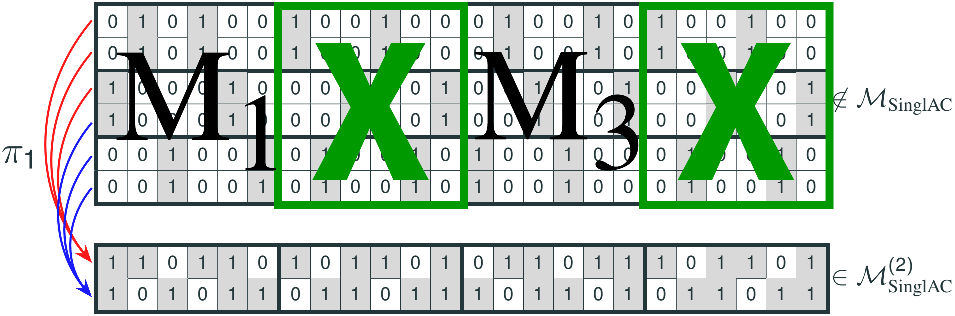
Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

$$\Sigma = \left\{ \begin{pmatrix} 0&1&0&1&0&0\\0&1&0&1&0&0\\1&0&0&0&1&0\\1&0&0&0&1&0\\0&0&1&0&0&1\\0&0&1&0&0&1 \end{pmatrix}, \begin{pmatrix} 1&0&0&1&0&0\\1&0&0&1&0&0\\0&0&1&0&0&1\\0&0&1&0&0&1\\0&1&0&0&1&0\\0&1&0&0&1&0 \end{pmatrix}, \begin{pmatrix} 0&1&0&0&1&0\\0&1&0&0&1&0\\0&0&1&0&0&1\\0&0&1&0&0&1\\1&0&0&1&0&0\\1&0&0&1&0&0 \end{pmatrix}, \begin{pmatrix} 1&0&0&1&0&0\\1&0&0&1&0&0\\0&0&1&0&0&1\\0&0&1&0&0&1\\0&1&0&0&1&0\\0&1&0&0&1&0 \end{pmatrix}, \ldots \right\}, \; C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}.$$

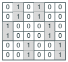
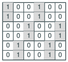
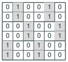
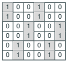
$$f\colon \Sigma^N \to C,\ f(M_1, \ldots, M_N) = (\xi(M_1|\ldots|M_N)^{\pi_1}, \ldots, \xi(M_1|\ldots|M_N)^{\pi_{12}})$$

By the Hales-Jewett Theorem, $f(M_1, X, M_3, X, \ldots)$ does not depend on X.

Minion homomorphism $\xi\colon \mathcal{M}_{\text{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.



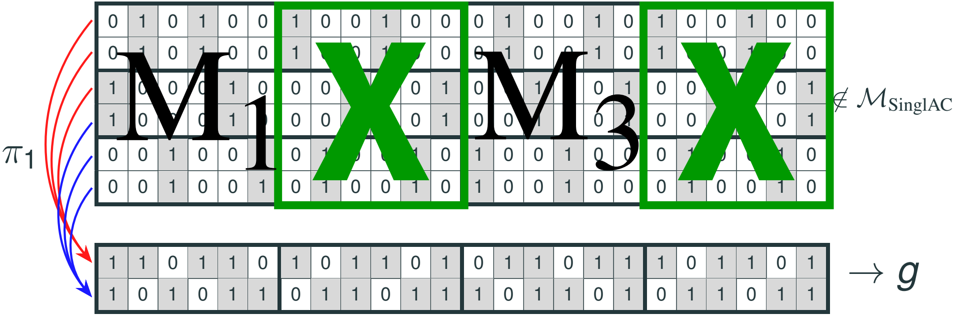
Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

$\Sigma = \{$ , , , , $\ldots \}$, $C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}$.

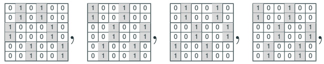
$f\colon \Sigma^N \to C$, $f(M_1, \ldots, M_N) = (\xi(M_1|\ldots|M_N)^{\pi_1}, \ldots, \xi(M_1|\ldots|M_N)^{\pi_{12}})$

By the Hales-Jewett Theorem, $f(M_1, X, M_3, X, \ldots)$ does not depend on X.

Minion homomorphism $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.



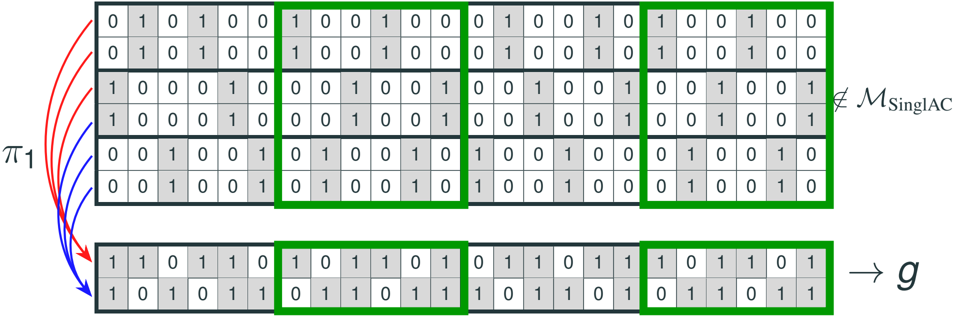
Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

$\Sigma = \{$  $, \ldots \}$, $C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}$.

$f\colon \Sigma^N \to C$, $f(M_1, \ldots, M_N) = (\xi(M_1|\ldots|M_N)^{\pi_1}, \ldots, \xi(M_1|\ldots|M_N)^{\pi_{12}})$

By the Hales-Jewett Theorem, $f(M_1, X, M_3, X, \ldots)$ does not depend on X.

Minion homomorphism $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.



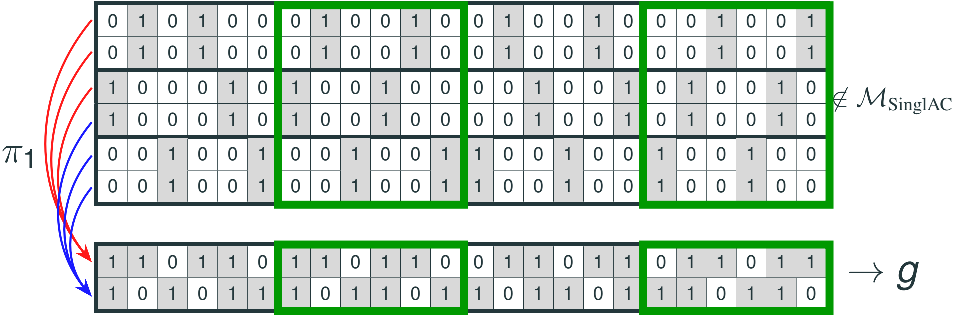
Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

$$\Sigma = \left\{ \begin{smallmatrix} 0&1&0&1&0&0\\ 0&1&0&1&0&0\\ 1&0&0&0&1&0\\ 1&0&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1 \end{smallmatrix}, \begin{smallmatrix} 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0 \end{smallmatrix}, \begin{smallmatrix} 0&1&0&0&1&0\\ 0&1&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 1&0&0&1&0&0\\ 1&0&0&1&0&0 \end{smallmatrix}, \begin{smallmatrix} 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0 \end{smallmatrix}, \ldots \right\}, C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}.$$

$$f\colon \Sigma^N \to C,\ f(M_1, \ldots, M_N) = (\xi(M_1|\ldots|M_N)^{\pi_1}, \ldots, \xi(M_1|\ldots|M_N)^{\pi_{12}})$$

By the Hales-Jewett Theorem, $f(M_1, X, M_3, X, \ldots)$ does not depend on X.

Minion homomorphism $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.



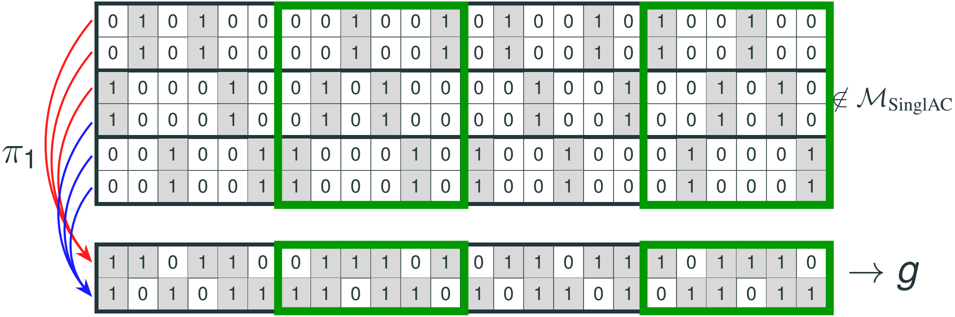
Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

$$\Sigma = \left\{ \begin{smallmatrix} 0&1&0&1&0&0\\ 0&1&0&1&0&0\\ 1&0&0&0&1&0\\ 1&0&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1 \end{smallmatrix}, \begin{smallmatrix} 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0 \end{smallmatrix}, \begin{smallmatrix} 0&1&0&0&1&0\\ 0&1&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 1&0&0&1&0&0\\ 1&0&0&1&0&0 \end{smallmatrix}, \begin{smallmatrix} 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0 \end{smallmatrix}, \ldots \right\}, \quad C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}.$$

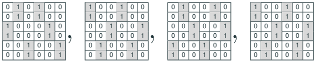
$$f\colon \Sigma^N \to C,\ f(M_1, \ldots, M_N) = (\xi(M_1|\ldots|M_N)^{\pi_1}, \ldots, \xi(M_1|\ldots|M_N)^{\pi_{12}})$$

By the Hales-Jewett Theorem, $f(M_1, X, M_3, X, \ldots)$ does not depend on X.

Minion homomorphism $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.



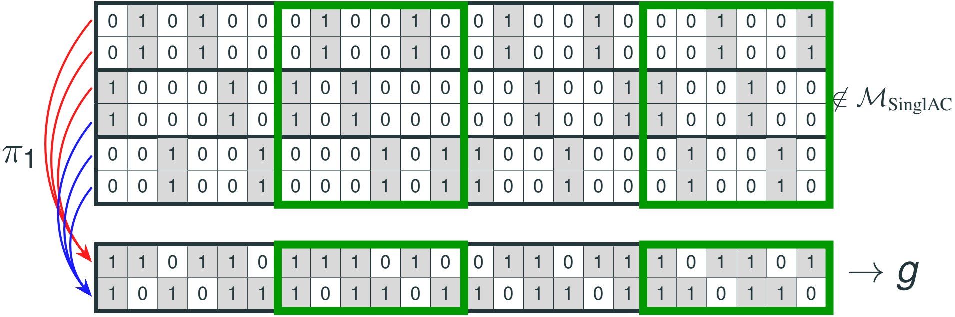
Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

$\Sigma = \{$  $, \ldots \}$, $C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}$.

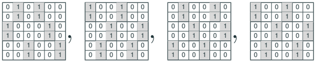
$f\colon \Sigma^N \to C$, $f(M_1, \ldots, M_N) = (\xi(M_1|\ldots|M_N)^{\pi_1}, \ldots, \xi(M_1|\ldots|M_N)^{\pi_{12}})$

By the Hales-Jewett Theorem, $f(M_1, X, M_3, X, \ldots)$ does not depend on X.

Minion homomorphism $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.



Minor maps: $\pi_1, \pi_2, \pi_3, \dots, \pi_{12}$.

$\Sigma = \{$  $, \dots \}$, $C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}$.

$f\colon \Sigma^N \to C$, $f(M_1, \dots, M_N) = (\xi(M_1|\dots|M_N)^{\pi_1}, \dots, \xi(M_1|\dots|M_N)^{\pi_{12}})$

By the Hales-Jewett Theorem, $f(M_1, X, M_3, X, \dots)$ does not depend on X.

Minion homomorphism $\xi\colon \mathcal{M}_{\mathrm{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.

π_1

0	1	0	1	0	0	1	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0
0	1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0
1	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0
1	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	0	0
0	0	1	0	0	1	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	1	0
0	0	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1

$\in \mathcal{M}_{\mathrm{SinglAC}}$

1	1	0	1	1	0	1	1	1	0	0	0	0	1	1	0	1	1	1	1	1	0	0	0
1	0	1	0	1	1	0	0	0	1	1	1	1	0	1	1	0	1	0	0	0	1	1	1

$\to g$

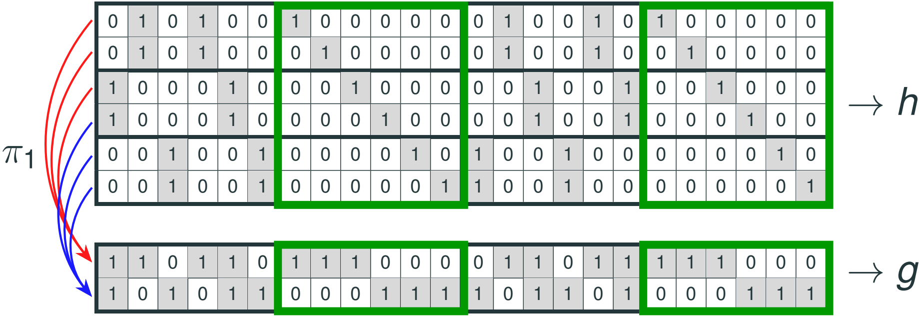
Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

$$\Sigma = \left\{ \begin{pmatrix} 0&1&0&1&0&0\\ 0&1&0&1&0&0\\ 1&0&0&0&1&0\\ 1&0&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1 \end{pmatrix}, \begin{pmatrix} 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0 \end{pmatrix}, \begin{pmatrix} 0&1&0&0&1&0\\ 0&1&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 1&0&0&1&0&0\\ 1&0&0&1&0&0 \end{pmatrix}, \begin{pmatrix} 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0 \end{pmatrix}, \ldots \right\}, \; C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}.$$

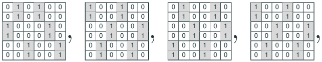
$$f\colon \Sigma^N \to C,\; f(M_1, \ldots, M_N) = (\xi(M_1|\ldots|M_N)^{\pi_1}, \ldots, \xi(M_1|\ldots|M_N)^{\pi_{12}})$$

By the Hales-Jewett Theorem, $f(M_1, X, M_3, X, \ldots)$ does not depend on X.

Minion homomorphism $\xi\colon \mathcal{M}_{\text{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.



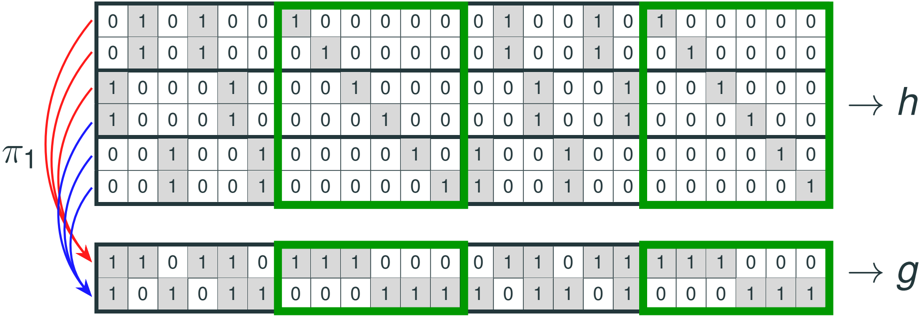
Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

$\Sigma = \{$  $, \ldots \}$, $C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}$.

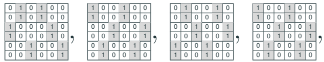
$f\colon \Sigma^N \to C$, $f(M_1, \ldots, M_N) = (\xi(M_1|\ldots|M_N)^{\pi_1}, \ldots, \xi(M_1|\ldots|M_N)^{\pi_{12}})$

By the Hales-Jewett Theorem, $f(M_1, X, M_3, X, \ldots)$ does not depend on X.

Minion homomorphism $\xi\colon \mathcal{M}_{\text{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.



Minor maps: $\pi_1, \pi_2, \pi_3, \dots, \pi_{12}$.

$\Sigma = \Big\{$  $, \dots \Big\}$, $C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}$.

$f\colon \Sigma^N \to C$, $f(M_1, \dots, M_N) = (\xi(M_1|\dots|M_N)^{\pi_1}, \dots, \xi(M_1|\dots|M_N)^{\pi_{12}})$

By the Hales-Jewett Theorem, $f(M_1, X, M_3, X, \dots)$ does not depend on X.

$h(x, x, x, y, y, y)$

Minion homomorphism $\xi\colon \mathcal{M}_{\text{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.

π_2

0	1	0	1	0	0	1	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0
0	1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0
1	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0
1	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	0	0
0	0	1	0	0	1	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	1	0
0	0	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1

$\to h$

1	1	0	1	1	0	1	1	0	1	0	0	0	1	1	0	1	1	1	1	0	1	0	0
1	0	1	0	1	1	0	0	1	0	1	1	1	0	1	1	0	1	0	0	1	0	1	1

$\to g$

Minor maps: $\pi_1, \pi_2, \pi_3, \ldots, \pi_{12}$.

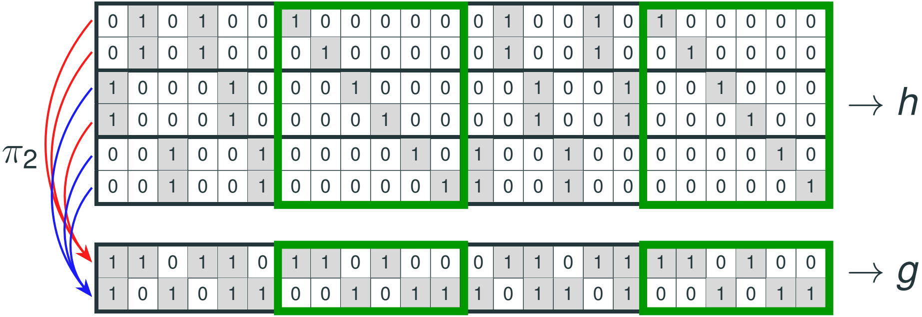
$$\Sigma = \left\{ \begin{array}{|c|c|c|c|c|c|}\hline 0&1&0&1&0&0\\ 0&1&0&1&0&0\\ 1&0&0&0&1&0\\ 1&0&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ \hline\end{array}, \begin{array}{|c|c|c|c|c|c|}\hline 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0\\ \hline\end{array}, \begin{array}{|c|c|c|c|c|c|}\hline 0&1&0&0&1&0\\ 0&1&0&0&1&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ \hline\end{array}, \begin{array}{|c|c|c|c|c|c|}\hline 1&0&0&1&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&0&1\\ 0&0&1&0&0&1\\ 0&1&0&0&1&0\\ 0&1&0&0&1&0\\ \hline\end{array}, \ldots \right\}, \; C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}.$$

$$f\colon \Sigma^N \to C,\ f(M_1, \ldots, M_N) = (\xi(M_1|\ldots|M_N)^{\pi_1}, \ldots, \xi(M_1|\ldots|M_N)^{\pi_{12}})$$

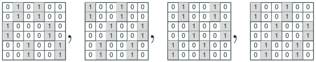
By the Hales-Jewett Theorem, $f(M_1, X, M_3, X, \ldots)$ does not depend on X.

$h(x, x, x, y, y, y) =$

Minion homomorphism $\xi\colon \mathcal{M}_{\text{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.



Minor maps: $\pi_1, \pi_2, \pi_3, \dots, \pi_{12}$.

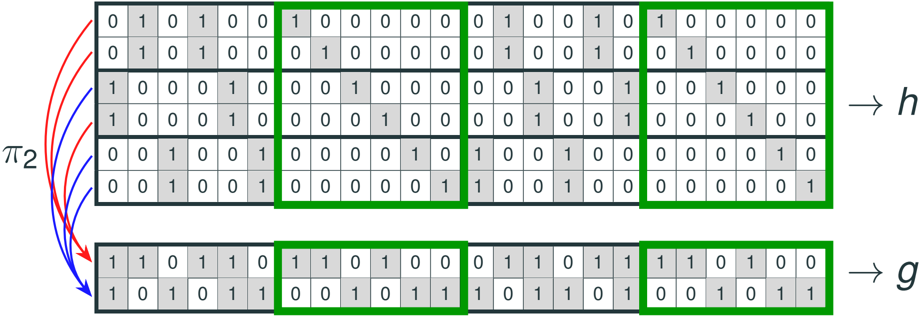
$\Sigma = \Big\{$  $, \dots \Big\}$, $C = \underbrace{\mathcal{N}^{(2)} \times \cdots \times \mathcal{N}^{(2)}}_{12}$.

$f\colon \Sigma^N \to C$, $f(M_1, \dots, M_N) = (\xi(M_1|\dots|M_N)^{\pi_1}, \dots, \xi(M_1|\dots|M_N)^{\pi_{12}})$

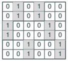
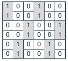
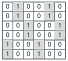
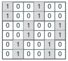
By the Hales-Jewett Theorem, $f(M_1, X, M_3, X, \dots)$ does not depend on X.

$h(x, x, x, y, y, y) = h(x, x, y, x, y, y)$

Minion homomorphism $\xi\colon \mathcal{M}_{\text{SinglAC}} \to \mathcal{N}$, where $\mathcal{N}$ is locally finite.



Minor maps: $\pi_1, \pi_2, \pi_3, \dots, \pi_{12}$.

$\Sigma = \{$ , , , , $\dots \}$, $C = \underbrace{\mathcal{N}^{(2)} \times \dots \times \mathcal{N}^{(2)}}_{12}$.

$f\colon \Sigma^N \to C$, $f(M_1, \dots, M_N) = (\xi(M_1|\dots|M_N)^{\pi_1}, \dots, \xi(M_1|\dots|M_N)^{\pi_{12}})$

By the Hales-Jewett Theorem, $f(M_1, X, M_3, X, \dots)$ does not depend on X.

$h(x, x, x, y, y, y) = h(x, x, y, x, y, y)$, $h(x, y, x, x, y, y) = h(y, x, x, x, y, y)$

Finiteness is crucial!

Finiteness is crucial!

Palette functions

Finiteness is crucial!

Palette functions

Finiteness is crucial!

Palette functions

A function is $(2, 2, 2)$-palette symmetric if

Finiteness is crucial!

Palette functions

A function is (2, 2, 2)-palette symmetric if

$$f(\underbrace{x,y},\underbrace{x,x},\underbrace{y,y}) = f(\underbrace{y,x},\underbrace{x,x},\underbrace{y,y})$$

$$f(\underbrace{x,x},\underbrace{x,y},\underbrace{y,y}) = f(\underbrace{x,x},\underbrace{y,x},\underbrace{y,y})$$

$$f(\underbrace{x,x},\underbrace{y,y},\underbrace{x,y}) = f(\underbrace{x,x},\underbrace{y,y},\underbrace{y,x})$$

Finiteness is crucial!

Palette functions

A function is (2, 2, 2)-palette symmetric if

$$f(\underbrace{x,y},\underbrace{x,x},\underbrace{y,y}) = f(\underbrace{y,x},\underbrace{x,x},\underbrace{y,y})$$

$$f(\underbrace{x,x},\underbrace{x,y},\underbrace{y,y}) = f(\underbrace{x,x},\underbrace{y,x},\underbrace{y,y})$$

$$f(\underbrace{x,x},\underbrace{y,y},\underbrace{x,y}) = f(\underbrace{x,x},\underbrace{y,y},\underbrace{y,x})$$

Theorem

Suppose $\mathbb{A}, \mathbb{B}$ are finite structures. TFAE

Finiteness is crucial!

Palette functions

A function is (2, 2, 2)-palette symmetric if

$$f(\underbrace{x,y},\underbrace{x,x},\underbrace{y,y}) = f(\underbrace{y,x},\underbrace{x,x},\underbrace{y,y})$$

$$f(\underbrace{x,x},\underbrace{x,y},\underbrace{y,y}) = f(\underbrace{x,x},\underbrace{y,x},\underbrace{y,y})$$

$$f(\underbrace{x,x},\underbrace{y,y},\underbrace{x,y}) = f(\underbrace{x,x},\underbrace{y,y},\underbrace{y,x})$$

Theorem

Suppose $\mathbb{A}, \mathbb{B}$ are finite structures. TFAE

1. Constraint Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.

Finiteness is crucial!

Palette functions

A function is (2, 2, 2)-palette symmetric if

$$f(\underbrace{x,y},\underbrace{x,x},\underbrace{y,y}) = f(\underbrace{y,x},\underbrace{x,x},\underbrace{y,y})$$

$$f(\underbrace{x,x},\underbrace{x,y},\underbrace{y,y}) = f(\underbrace{x,x},\underbrace{y,x},\underbrace{y,y})$$

$$f(\underbrace{x,x},\underbrace{y,y},\underbrace{x,y}) = f(\underbrace{x,x},\underbrace{y,y},\underbrace{y,x})$$

Theorem

Suppose $\mathbb{A}, \mathbb{B}$ are finite structures. TFAE

1. Constraint Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.
2. Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.

Finiteness is crucial!

Palette functions

A function is $(2,2,2)$-palette symmetric if

$$f(\underbrace{x,y},\underbrace{x,x},\underbrace{y,y}) = f(\underbrace{y,x},\underbrace{x,x},\underbrace{y,y})$$

$$f(\underbrace{x,x},\underbrace{x,y},\underbrace{y,y}) = f(\underbrace{x,x},\underbrace{y,x},\underbrace{y,y})$$

$$f(\underbrace{x,x},\underbrace{y,y},\underbrace{x,y}) = f(\underbrace{x,x},\underbrace{y,y},\underbrace{y,x})$$

Theorem

Suppose $\mathbb{A}, \mathbb{B}$ are finite structures. TFAE

1. Constraint Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.
2. Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.
3. $\mathrm{Pol}(\mathbb{A}, \mathbb{B})$ contains $(\underbrace{n, \ldots, n}_{m})$-palette symmetric function for all m, n.

Finiteness is crucial!

Theorem

Suppose $\mathbb{A}, \mathbb{B}$ are finite structures. TFAE

1. Constraint Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.
2. Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.
3. $\mathrm{Pol}(\mathbb{A}, \mathbb{B})$ contains $(\underbrace{n, \dots, n}_{m})$-palette symmetric function for all m, n.

Finiteness is crucial!

Theorem

Suppose $\mathbb{A}, \mathbb{B}$ are finite structures. TFAE

1. Constraint Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.
2. Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.
3. $\mathrm{Pol}(\mathbb{A}, \mathbb{B})$ contains $(\underbrace{n, \ldots, n}_{m})$-palette symmetric function for all m, n.

Lemma

Suppose $\mathbb{F} = (\mathbb{Q}; x \neq y, x = y \Rightarrow u = v)$.

Finiteness is crucial!

Theorem

Suppose $\mathbb{A}, \mathbb{B}$ are finite structures. TFAE

1. Constraint Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.
2. Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.
3. $\mathrm{Pol}(\mathbb{A}, \mathbb{B})$ contains $(\underbrace{n, \ldots, n}_{m})$-palette symmetric function for all m, n.

Lemma

Suppose $\mathbb{F} = (\mathbb{Q}; x \neq y, x = y \Rightarrow u = v)$. Then

1. Constraint Singleton BLP solves $\mathrm{PCSP}(\mathbb{F}|_{\{1,2,\ldots,n\}}, \mathbb{F})$.

Finiteness is crucial!

Theorem

Suppose $\mathbb{A}, \mathbb{B}$ are finite structures. TFAE

1. Constraint Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.
2. Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.
3. $\mathrm{Pol}(\mathbb{A}, \mathbb{B})$ contains $(\underbrace{n, \ldots, n}_{m})$-palette symmetric function for all m, n.

Lemma

Suppose $\mathbb{F} = (\mathbb{Q}; x \neq y, x = y \Rightarrow u = v)$. Then

1. Constraint Singleton BLP solves $\mathrm{PCSP}(\mathbb{F}|_{\{1,2,\ldots,n\}}, \mathbb{F})$.
2. Singleton BLP **does not** solve $\mathrm{PCSP}(\mathbb{F}|_{\{1,2,\ldots,n\}}, \mathbb{F})$.

Finiteness is crucial!

Theorem

Suppose $\mathbb{A}, \mathbb{B}$ are finite structures. TFAE

1. Constraint Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.
2. Singleton BLP solves $\mathrm{PCSP}(\mathbb{A}, \mathbb{B})$.
3. $\mathrm{Pol}(\mathbb{A}, \mathbb{B})$ contains $(\underbrace{n, \ldots, n}_{m})$-palette symmetric function for all m, n.

Lemma

Suppose $\mathbb{F} = (\mathbb{Q}; x \neq y, x = y \Rightarrow u = v)$. Then

1. Constraint Singleton BLP solves $\mathrm{PCSP}(\mathbb{F}|_{\{1,2,\ldots,n\}}, \mathbb{F})$.
2. Singleton BLP **does not** solve $\mathrm{PCSP}(\mathbb{F}|_{\{1,2,\ldots,n\}}, \mathbb{F})$.
3. $\mathrm{Pol}(\mathbb{F}|_{\{1,2,\ldots,n\}}, \mathbb{F})$ **does not** contain $(n_1, \ldots, n_m)$-palette symmetric function for any $m, n_1, \ldots, n_m \geq 4$.


THANK
YOU
FOR
YOUR
ATTENTION

Results

Theorem [Zhuk, 2025]

Suppose $\mathbb{A}$ is a temporal structure. Then

- $\mathrm{PCSP}(\mathbb{A}|_{\{1,\ldots,N\}}, \mathbb{A})$ is solvable by CSINGLAC $\wedge$ $\mathrm{SinglAIP}$, or
- $\mathrm{CSP}(\mathbb{A})$ is NP-hard.

Results

Theorem [Zhuk, 2025]

Suppose $\mathbb{A}$ is a temporal structure. Then

- $\mathrm{PCSP}(\mathbb{A}|_{\{1,\dots,N\}}, \mathbb{A})$ is solvable by CSingLAC $\wedge$ $\mathrm{SinglAIP}$, or
- $\mathrm{CSP}(\mathbb{A})$ is NP-hard.

Theorem [Zhuk, 2025]

Suppose $\mathrm{CSP}(\mathbb{A})$ is in P, $|A| < 8$. Then

- $\mathrm{Pol}(\mathbb{A})$ contains a $(\underbrace{p,\dots,p}_{n})$-palette symmetric function for every prime $p > 7$ and $n \in \mathbb{N}$.
- $\mathrm{Singl}(\mathrm{BLP} + \mathrm{AIP})$ solves $\mathrm{CSP}(\mathbb{A})$.