

DECIDABILITY OF INTERPRETABILITY

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CSPs $A = (A ; R_1, \dots, R_n)$ relational structure

CSP(A)

INPUT : finite rel. structure I

QUESTION : $I \xrightarrow{\text{hom.}} A ?$

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- every computational problem is P-time equiv. to CSP

(Bodirsky + Grohe 2008)

- P vs. NP-c. dichotomy for finite-domain CSPs + nice hardness criterion

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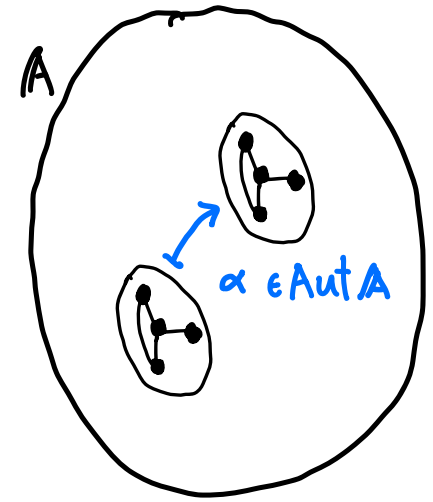
Conjecture: P vs. NP-c. dichotomy for fo-reducts of fbh structures + nice hardness criterion.

(Bodirsky + Pinsker 2011)

Finely bounded homogeneous (fbh)

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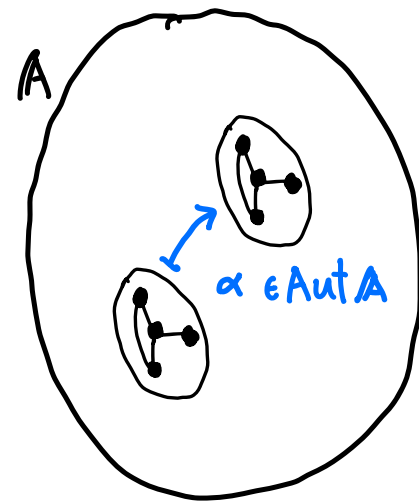
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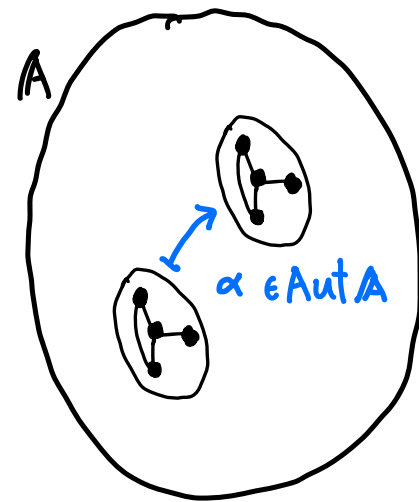
- **homogeneous** if isomorphism between finite substructures extend to automorphisms of A
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- $\forall$ finite C : C embeds into $A \iff$ no F_i embeds into C



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Examples • Random graph

bounds: $\mathbb{Q}$, $\bullet \rightarrow \bullet$

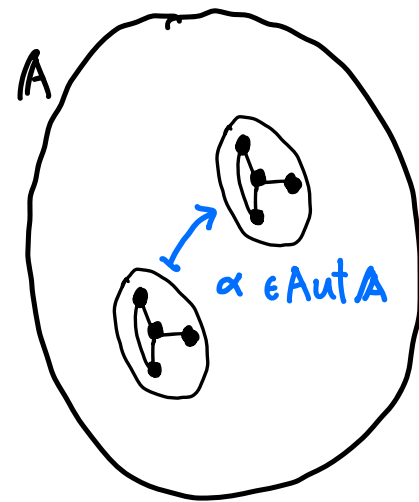
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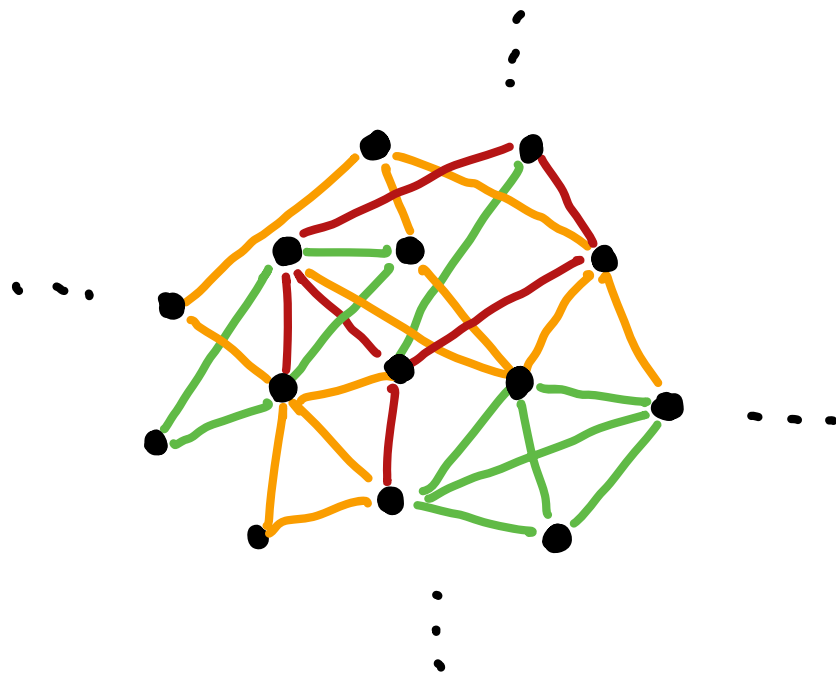
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Examples • Random graph

• $(\mathbb{Q}, <)$

• $\text{Flim} \left\{ \begin{array}{l} \text{edge-colored finite} \\ \text{-free graphs} \end{array} \right\}$



(Some) tools for the conjecture

Polymorphisms

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Model-complete cores

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 $e|_F \in \text{Aut } A|_F \quad \forall F \subseteq A \text{ finite.}$

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Question 2: How hard is it to find
 the model-complete core of A ?

Question 1A: "How hard to check if $\text{pol } A = \text{pol } B$?"

Thm. (Bodirsky + Pinsker + Tsankov 2011)

"Decidability of Definability"

If $A, B \leq_{fo} C$, C fph + Ramsey , then can decide if

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A, B are pp-interdefinable

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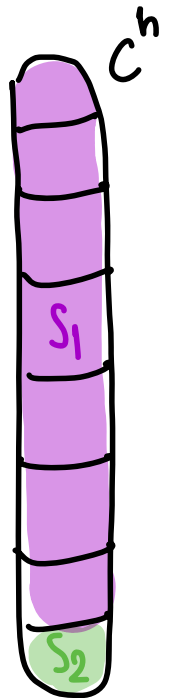
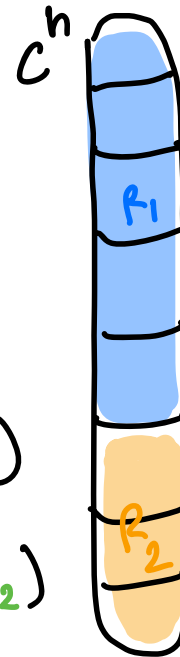
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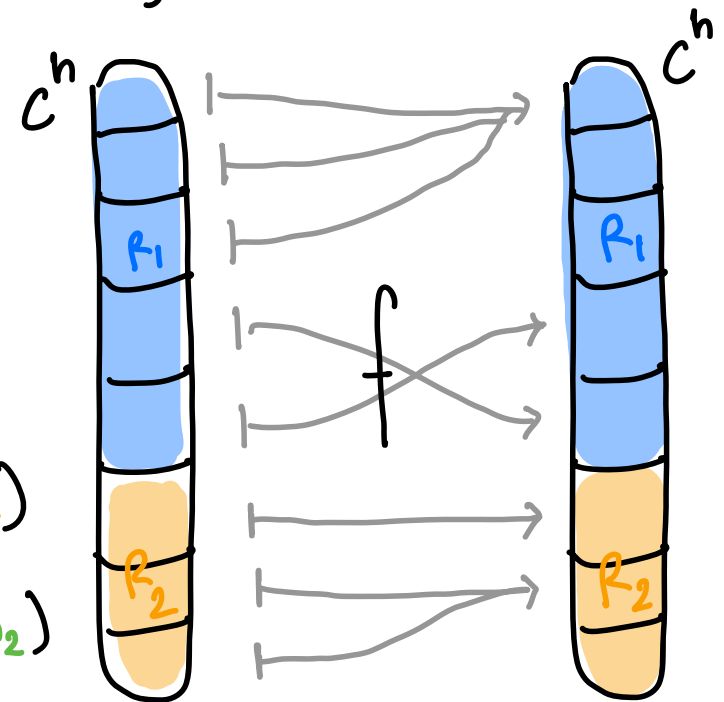
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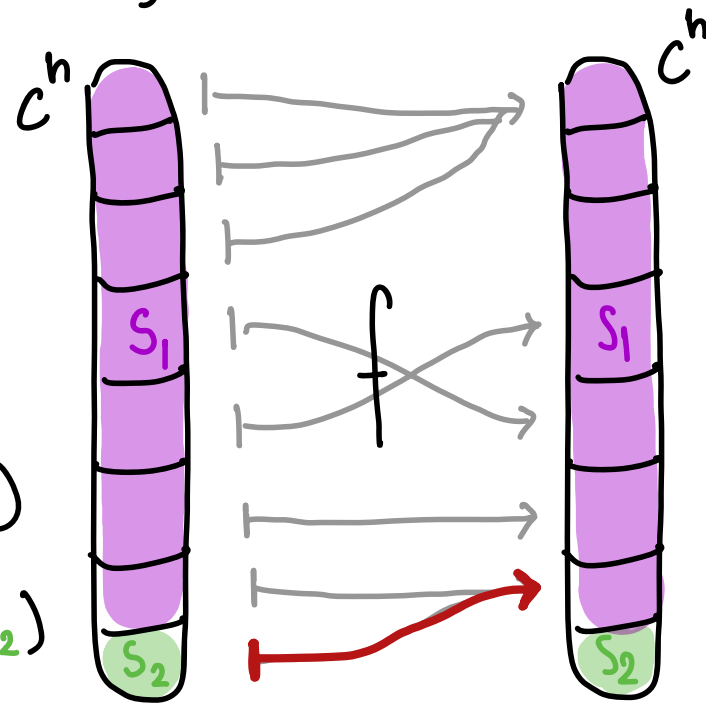
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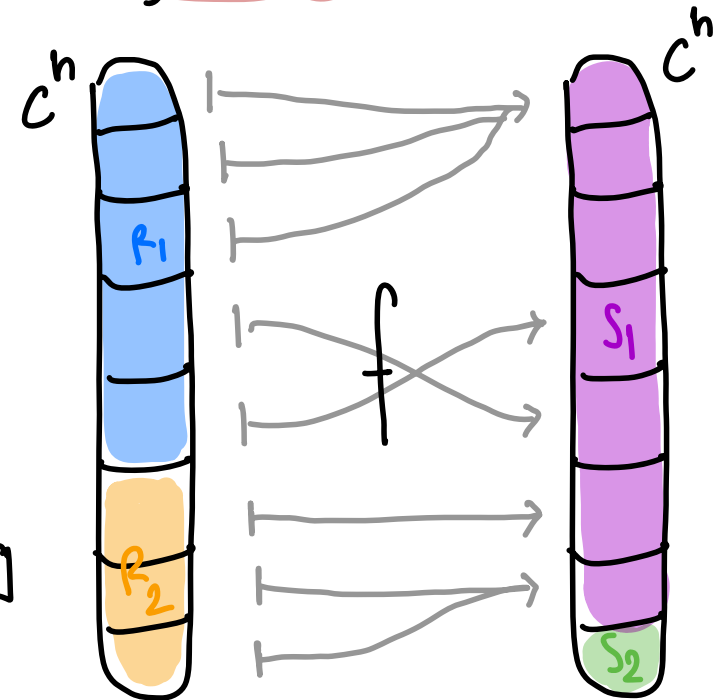
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fbh: for large enough n (computable)
can decide if operation on
 $(\text{Aut } C \curvearrowright C^n)$ -orbits is induced
by polymorphism of A or B . $\square$



Question 2: "How hard is it to find m-c core?"

5/8

Thm. [F. + Pinski 2026]

If $A \leq_{f_0} C$, C fbh + Ramsey, then can compute the m-c core A^c of A .

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C general: Choose $f \in \text{End } A$ acting on $(\text{Aut } C \curvearrowright C^n)$ -orbits $\forall n$ s.t. $f(A)$ intersects minimal set of orbits.

- $A^c = "f(A)"$

□

Question 1B: "How hard to check if $\text{Pol } A \cong_{\text{top}} \text{Pol } B$?"

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If $A \leq_{f_0} C$, $B \leq_{f_0} D$ and C, D f.b.h. + Ramsey, A^c, B^c without algebraicity, 1-transitive, then can decide if

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• (Rubin 1994) A, B ω -cat. without algebraicity:

in general only find interpret. of dim. n $T: A^n \cong U \rightarrow B$ for some n .

$$\text{Aut } A \cong_{\text{top}} \text{Aut } B \Leftrightarrow \exists \text{ bijection } \phi: A \rightarrow B$$

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• with additional assumption the same holds for $\text{Pol } A^c, \text{Pol } B^c$.

Beyond decidability : measuring complexity

7/8

- First-order reducts of : can be finitely represented
f.b.h. structures

makes sense to ask : "Is $\text{Pol } A \cong_{\text{f.o.}} \text{Pol } B$ decidable?"

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Equiv. rel. $\sim$ on Borel space X is smooth, if

exists $I : X \xrightarrow{\text{Borel}} \mathbb{R}$ s.t. $x \sim y \Leftrightarrow I(x) = I(y)$.

Smoothness of top. isomorphism

8/8

Thm. (Nies + Paolini 2024)

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Thm. (F. + Pinster 2026) $A \sim B : \Leftrightarrow \text{PoL } A \cong_{\text{top}} \text{PoL } B$ is smooth for ω -cat., 1-transitive model-complete cores without algebraicity.

Thank you !