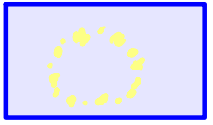
DECIDABILITY OF INTERPRETABILITY

MICHAEL PINSKER

TU WIEN

INFINITE STRUCTURAL RAMSEY THEORY, BIRS 2025



EUROPEAN RESEARCH COUNCIL

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FWF I5948

EQUIVALENCE RELATIONS ON CLASS OF ω-CATEGORICAL STRUCTURES GIVEN BY THEIR SPACES OF SYMMETRIES

EQUIVALENCE RELATIONS ON CLASS OF ω-CATEGORICAL STRUCTURES GIVEN BY THEIR SPACES OF SYMMETRIES

$\mathbb{A}$ COUNTABLE, ω-CATEGORICAL $:\Leftrightarrow$ $A^n/\mathrm{Aut}\,\mathbb{A}$ FINITE $\forall n$

EXAMPLE GENERIC LINEAR ORDER, POSET, GRAPH, K_n-FREE GRAPH, ...

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BI-DEFINABILITY / CONJUGACY / ACTION ISOMORPHISM	$\mathrm{Aut}\,\mathbb{A} = \varphi\,\mathrm{Aut}\,\mathbb{B}$: $\mathrm{Aut}\,\mathbb{A} = \varphi^{-1}\,\mathrm{Aut}\,\mathbb{B}\,\varphi$
BI-INTERPRETABILITY / TOPOLOGICAL ISO	$\mathrm{Aut}\,\mathbb{A} \cong^{top} \mathrm{Aut}\,\mathbb{B}$
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INTERDEFINABILITY / EQUALITY	$\mathrm{Aut}\,\mathbb{A} = \mathrm{Aut}\,\mathbb{B}$	**Ex.** THOMAS' CONJECTURE
BI-DEFINABILITY / CONJUGACY / ACTION ISOMORPHISM	$\mathrm{Aut}\,\mathbb{A} =^{\varphi} \mathrm{Aut}\,\mathbb{B}$: $\mathrm{Aut}\,\mathbb{A} = \varphi^{-1}\,\mathrm{Aut}\,\mathbb{B}\,\varphi$	**Ex.** OLIGOMORPHICITY
BI-INTERPRETABILITY / TOPOLOGICAL ISO	$\mathrm{Aut}\,\mathbb{A} \cong^{\mathrm{top}} \mathrm{Aut}\,\mathbb{B}$	**Ex.** RAMSEY PROPERTY
ALGEBRAIC ISOMORPHISM	$\mathrm{Aut}\,\mathbb{A} \cong^{\mathrm{alg}} \mathrm{Aut}\,\mathbb{B}$	

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COMPLEXITY? (DESCRIPTIVE / COMPUTATIONAL)

CONTEXT

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$\mathrm{Pol}(\mathbb{A}) := \{ f : \mathbb{A}^n \to \mathbb{A} \mid f \text{ HOMOMORPHISM} \}$

EXAMPLE MONOTONE MULTIVARIATE FUNCTIONS ON $(\mathbb{Q}, <)$

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PRIMITIVE POSITIVE FORMULA:

$\exists x_1 \dots \exists x_n \; R_1(\text{VARIABLES}) \wedge R_2(\;) \wedge \dots$

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GIVEN FINITE $\mathbb{B}$

QUESTION $\mathbb{B} \xrightarrow{\text{HOM}} \mathbb{A}$?

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- $\mathrm{CSP}(K_3)$: 3-COLORING
- $\mathrm{CSP}(\mathbb{Q}, <)$: DIGRAPH ACYCLICITY

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- $\mathrm{CSP}(\mathbb{Q}, <)$: DIGRAPH ACYCLICITY

- EVERY COMPUTATIONAL PROBLEM POLYNOMIAL-TIME EQUIVALENT TO SOME CSP (BODIRSKY + GROHE '08)

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- $\mathbb{A}$ ω-CATEGORICAL :

  $\mathrm{Pol}\,\mathbb{A} = \mathrm{Pol}\,\mathbb{B} \Rightarrow \mathrm{CSP}\,\mathbb{A} \sim \mathrm{CSP}\,\mathbb{B}$

  (BODIRSKY + NEŠETŘIL '03)

  $\cong^{\text{fo}}$

  (BODIRSKY + P. '12)

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$\cong^{\text{top}}$

(BODIRSKY + P. '12)

CONJECTURE (BODIRSKY + P. '11)

$\mathbb{A}$ FO-REDUCT OF $\mathbb{F} = \mathrm{Flim}(\mathrm{Forb}(\mathcal{S}))$

$\Rightarrow \mathrm{CSP}(\mathbb{A})$ $\in P$ * / NP-COMPLETE

* IF $\mathrm{Pol}\,\mathbb{A} \models \exists u, v, f$

$u \cdot f(x,y,x,z,y,z) =$

$v \cdot f(y,x,z,x,z,y)$

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- $\cong^{alg}$ vs. $\cong^{top}$: TOPOLOGICAL RECONSTRUCTION

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  - EVANS+ HEWITT '90 / BODIRSKY + EVANS + KOMPATSCHER + P. '15

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  - EVANS + HEWITT '90 / BODIRSKY + EVANS + KOMPATSCHER + P. '15

THEOREM (KNIES + TENT + SCHLICHT '22)
$\cong^{top}$ BOREL-REDUCES TO E_∞

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THEOREM (NIES+PAGNI '24, FELLER+P. '25)
$\cong^{top}$ SMOOTH : BOREL-REDUCES TO $=$
FOR $\mathbb{A}$ NO ALGEBRAICITY / SAP

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- $\cong^{\mathrm{alg}}$ vs. $\cong^{\mathrm{top}}$: TOPOLOGICAL RECONSTRUCTION
  ✗ EVANS + HEWITT '90 / BODIRSKY + EVANS + KOMPATSCHER + P. '15

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$\cong^{\mathrm{top}}$ BOREL-REDUCES TO E_∞

⇕ NO ALGEBRAICITY (RUBIN '94)

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DECIDABILITY

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FINITE PRESENTATION: $\mathbb{A}$ FO-REDUCT OF $\tilde{\mathbb{A}} = \mathrm{Flim}(\mathrm{Forb}(\mathcal{A}))$
$\mathbb{B}$ $\tilde{\mathbb{B}}$ $\mathcal{B}$

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INPUT: $\mathcal{A}$, $\mathcal{B}$, QF-FORMULAS DEFINING $\mathbb{A}$, $\mathbb{B}$

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THEOREM (BODIRSKY + P. + TSANKOV '11)

DECIDABLE: GIVEN $\mathbb{A}, \mathbb{B}$ WITH $\tilde{\mathbb{A}}, \tilde{\mathbb{B}}$ RAMSEY

QUESTION $\mathrm{End}\,\mathbb{A} = \mathrm{End}\,\mathbb{B}$?

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OPEN PROBLEM: $\mathrm{Aut}\,\mathbb{A} = \mathrm{Aut}\,\mathbb{B}$?

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OPEN PROBLEM: $\check{\mathbb{A}}, \check{\mathbb{B}}$ NOT RAMSEY ?

USEFUL?

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"PROOF" $\mathrm{Pol}(\mathbb{A}) \not\subseteq \mathrm{Pol}(\mathbb{B}) \iff \exists f \in \mathrm{Pol}(\mathbb{A}) \setminus \mathrm{Pol}(\mathbb{B})$

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"PROOF" $\mathrm{Pol}\mathbb{A} \not\subseteq \mathrm{Pol}\mathbb{B} \iff \exists f \in \mathrm{Pol}\mathbb{A} \setminus \mathrm{Pol}\mathbb{B}$

$\iff \exists f$ —··— , f $(\mathrm{Aut}\tilde{\mathbb{A}}, \mathrm{Aut}\tilde{\mathbb{B}})$-

CANONICAL

→ ACTS ON ORBITS

USEFUL?

THEOREM (BODIRSKY + P. + TSANKOV '11)

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→ ACTS ON ORBITS

THOMAS' CONJECTURE:

VERIFIED FOR GENERIC LINEAR ORDER, GRAPH, POSET, DIGRAPH, HYPERGRAPHS, ...

CAMERON '90 / THOMAS 91' / PACH + P. + PLUHÁR + PONGRÁCZ + SZABÓ '13 / AGARWAL + KOMPATSCHER '07 / THOMAS '96

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CSP COMPLEXITY CLASSIFICATIONS:

FO-REDUCTS OF:

- $(\mathbb{Q}, <)$ — BODIRSKY + KÁRA '07
- HOMOGENEOUS GRAPHS — BOD. + P '11 / BOD. + MARTIN + P. + PONGRÁCZ '15
- GENERIC POSET — KOMPATSCHER + VAN PHAM '16
- TOURNAMENT — MOTTET + P. '21
- HYPERGRAPH — MOTTET + NAGY + P. '23

...

	FIRST-ORDER	EXISTENTIAL POSITIVE	PRIMITIVE POSITIVE
INTERDEFINABILITY / EQUALITY	$\mathrm{Aut}\,\mathbb{A} = \mathrm{Aut}\,\mathbb{B}$	$\mathrm{End}\,\mathbb{A} = \mathrm{End}\,\mathbb{B}$	$\mathrm{Pol}\,\mathbb{A} = \mathrm{Pol}\,\mathbb{B}$
BI-DEFINABILITY / CONJUGACY / ACTION ISOMORPHISM	$\mathrm{Aut}\,\mathbb{A} = \varphi\,\mathrm{Aut}\,\mathbb{B}$: $\mathrm{Aut}\,\mathbb{A} = \varphi^{-1}\,\mathrm{Aut}\,\mathbb{B}\,\varphi$	$\mathrm{End}\,\mathbb{A} = \varphi\,\mathrm{End}\,\mathbb{B}$	$\mathrm{Pol}\,\mathbb{A} = \varphi\,\mathrm{Pol}\,\mathbb{B}$
BI-INTERPRETABILITY / TOPOLOGICAL ISO	$\mathrm{Aut}\,\mathbb{A} \cong^{top} \mathrm{Aut}\,\mathbb{B}$	$\mathrm{End}\,\mathbb{A} \cong^{top} \mathrm{End}\,\mathbb{B}$	$\mathrm{Pol}\,\mathbb{A} \cong^{top} \mathrm{Pol}\,\mathbb{B}$
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EQUALITY TOO FINE!

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CONJECTURE (Bodirsky + P. '11)

$\mathbb{A}$ FO-REDUCT OF $\mathbb{F} = \mathrm{Flim}(\mathrm{Forb}(\mathcal{F}))$

$\Rightarrow$ $\mathrm{CSP}(\mathbb{A})$ $\in$ P *

$\sim$ NP-COMPLETE

* IF $\mathrm{Pol}(\mathbb{A}) \models \exists u, v, f$

$u \circ f(x,y,x,z,y,z) =$

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EQUALITY TOO FINE!

THM (Rydval + Schnöbi + Piggs + P. '25)

CONJ. EQUIVALENT TO RESTRICTION TO NO ALGEBRAICITY

CONJECTURE (Bodirsky + P. '11)

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DEFINITION

$\mathbb{A}$ MODEL-COMPLETE CORE $:\Leftrightarrow$ $\overline{\mathrm{Aut}\,\mathbb{A}} = \mathrm{End}\,\mathbb{A}$

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THEOREM (BODIRSKY '05 / MOTTET+P. '21 / FELLER+P. '25)

- $\forall\, \mathbb{A}$ ω-CAT. $\exists\, \mathbb{A}^c \leq \mathbb{A}$ MC-CORE : $\mathrm{CSP}(\mathbb{A}') = \mathrm{CSP}(\mathbb{A})$
  $\mathbb{A}^c$ UNIQUE UP TO ISO, ω-CAT.
- $\mathbb{A}$ FO-REDUCT OF $\tilde{\mathbb{A}} = \mathrm{Flim}(\mathrm{Forb}(\mathcal{A}))$ RAMSEY $\Rightarrow$ SAME FOR $\mathbb{A}^c$
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THEOREM (FELLER+P. LATE '25)

DECIDABLE : GIVEN $\mathbb{A}, \mathbb{B}$ FO-REDUCT OF $\tilde{\mathbb{A}}, \tilde{\mathbb{B}}$ RAMSEY, NO ALGEBRAICITY

QUESTION
- $\mathrm{Pol}(\mathbb{A}^c) \cong^{\mathrm{top}} \mathrm{Pol}(\mathbb{B}^c)$?
- $\mathrm{End}(\mathbb{A}^c) \cong^{\mathrm{top}} \mathrm{End}(\mathbb{B}^c)$?
- $\mathrm{Aut}(\mathbb{A}^c) \cong^{\mathrm{top}} \mathrm{Aut}(\mathbb{B}^c)$?

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Thank you!

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