Kőnig = Ramsey,
A compactness lemma for Ramsey categories

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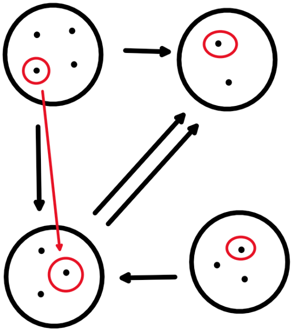
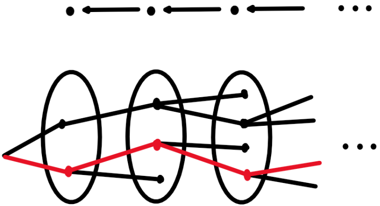
INPUT: some sets and maps



TASK: pick one element out of each set, compatibly.

Kőnig's tree lemma

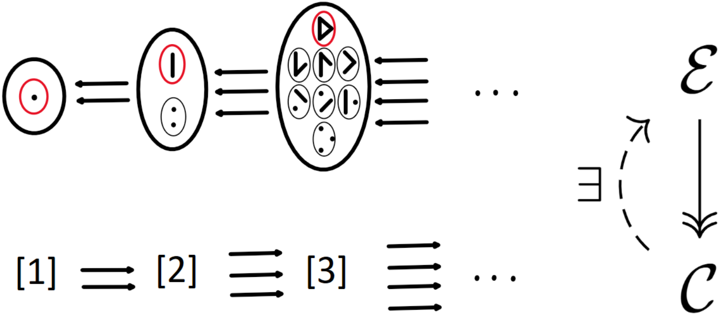
Shape:



if sets are **finite** and **nonempty** $\implies \exists$ solution

THM (H): Let $\mathcal{C}$ be a category*. Then:

König's lemma holds for $\mathcal{C}$ $\iff$ $\mathcal{C}^{\mathrm{op}}$ has JEP* and is Ramsey



Diagrams $\leftrightarrow$ **Expansions:** a Ramsey expansion is "optimal" iff its a core w.r.t. homomorphisms of diagrams.

Opfibrations: alg. top. inspired Ramsey transfer that generalizes

- products
- adding constants
- blowups
- parts of partite construction

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Thank you!