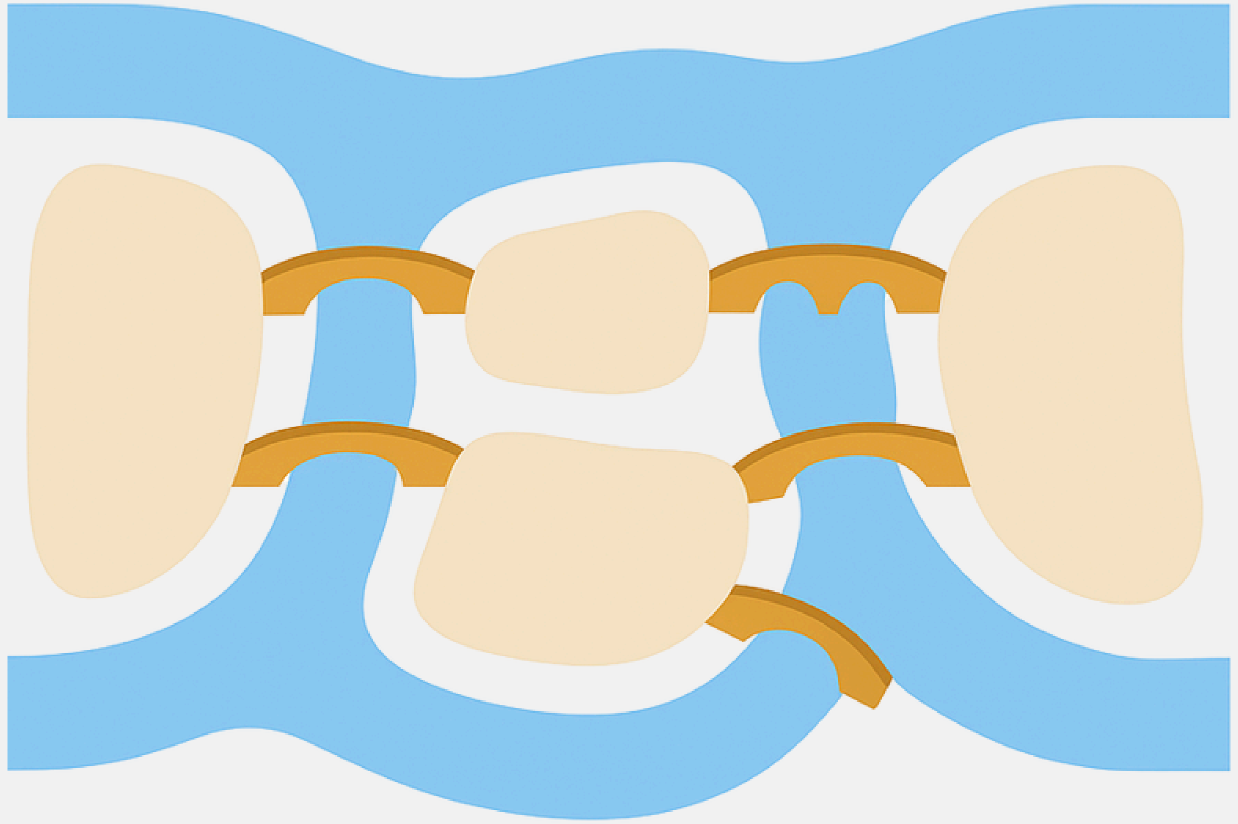
GRAPHS ARE EVERYWHERE

From the seven bridges of Königsberg to pattern recognition



European Research Council
Established by the European Commission

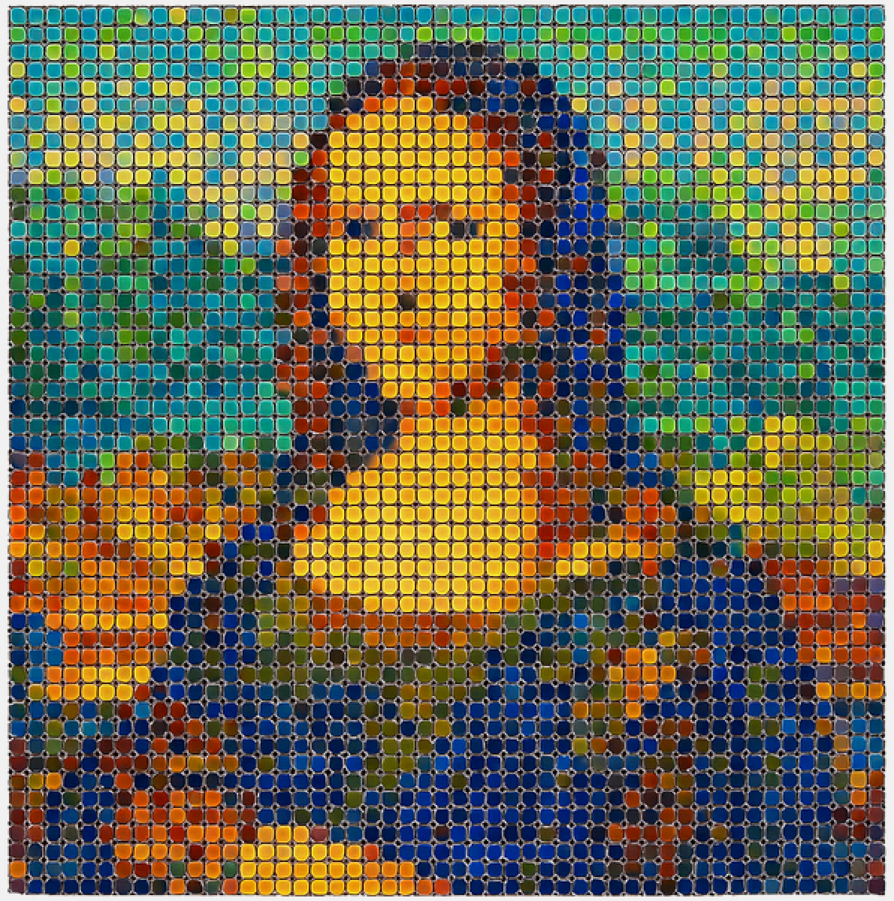
ERC Synergy Grant POCOCOP (GA 101071674)

WHERE TO FIND THEM?

Social Network

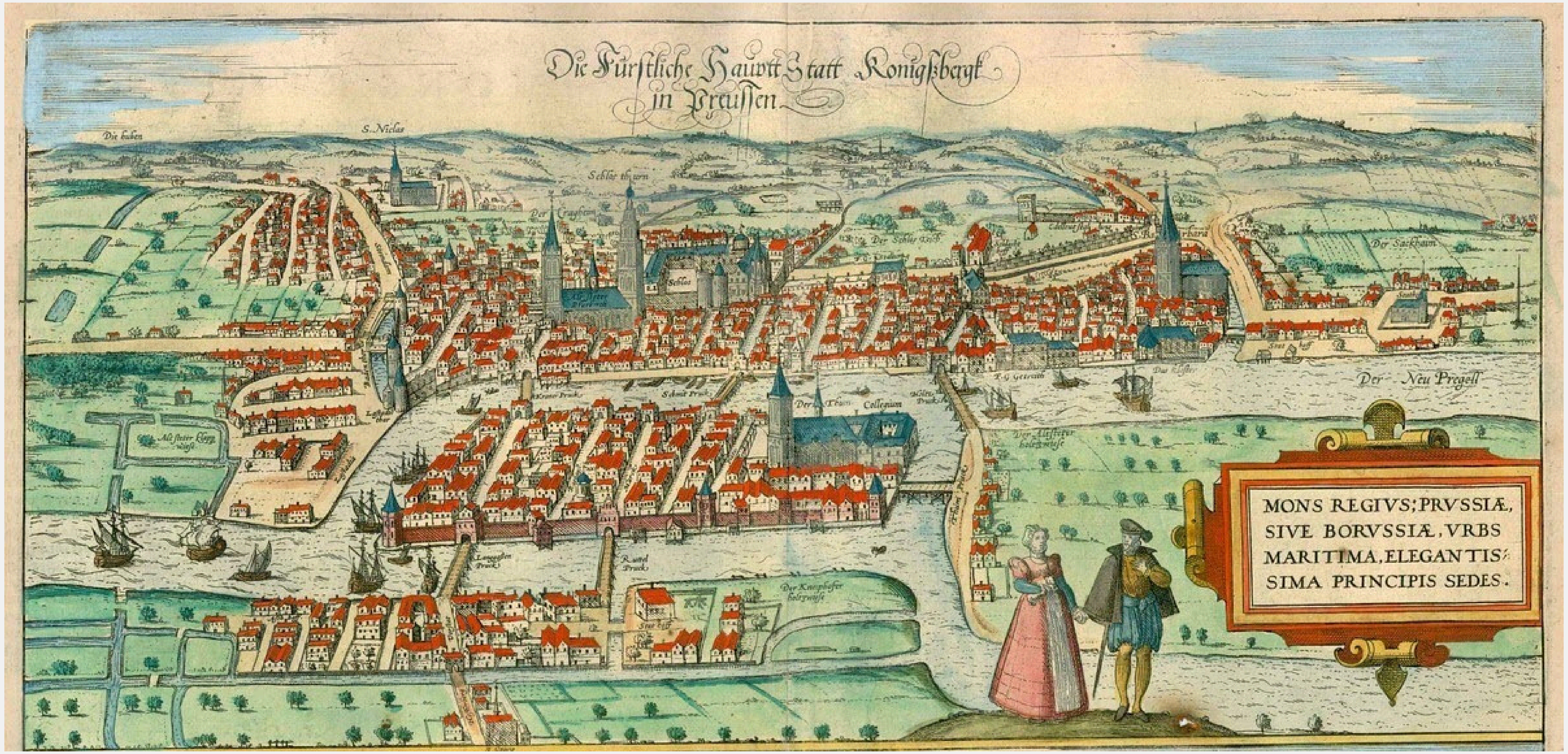
Subway map

Computer Images



THE PUZZLE THAT STARTED IT ALL

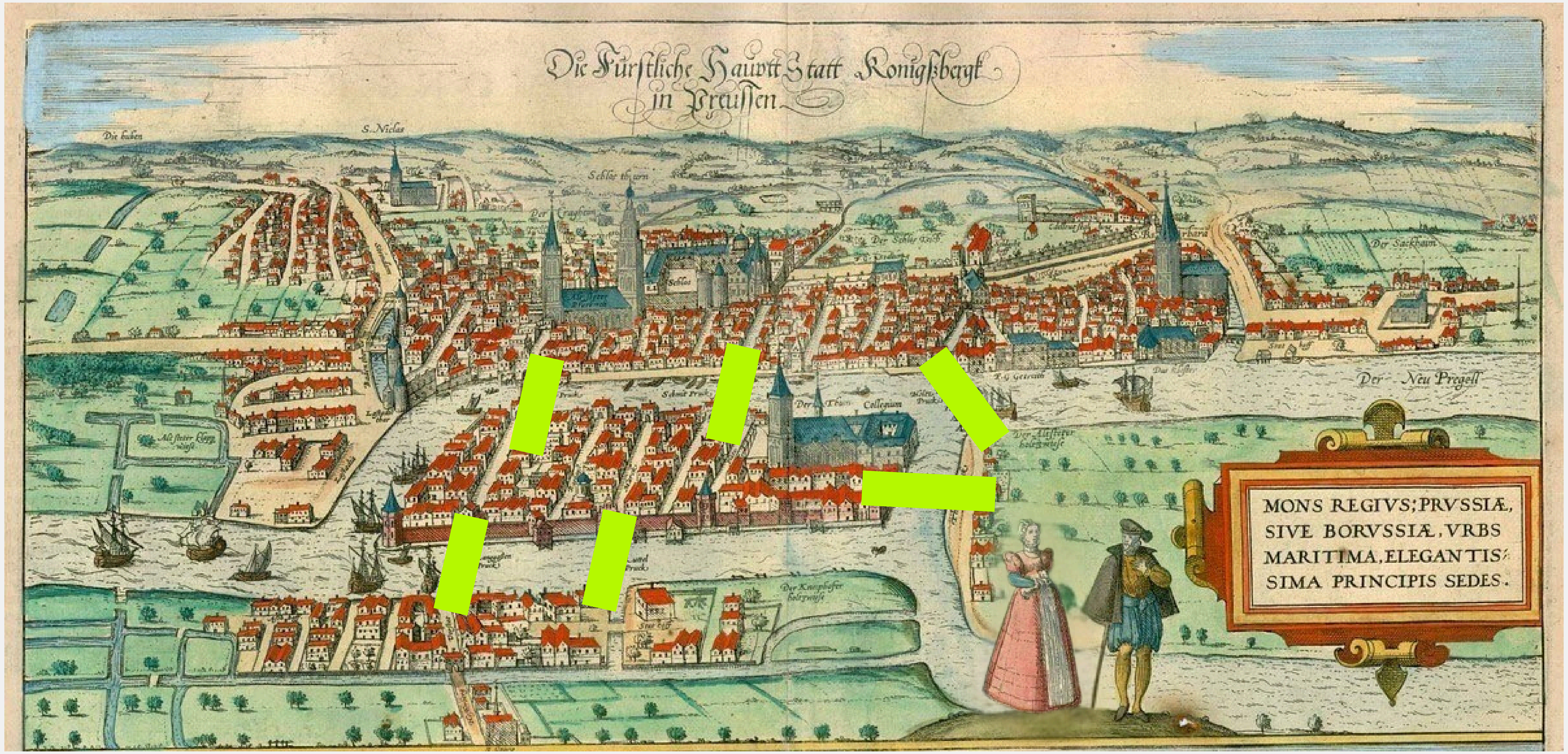
The seven bridges of Königsberg



Source: https://www.amusingplanet.com/2018/08/the-seven-bridges-of-konigsberg.html

THE PUZZLE THAT STARTED IT ALL

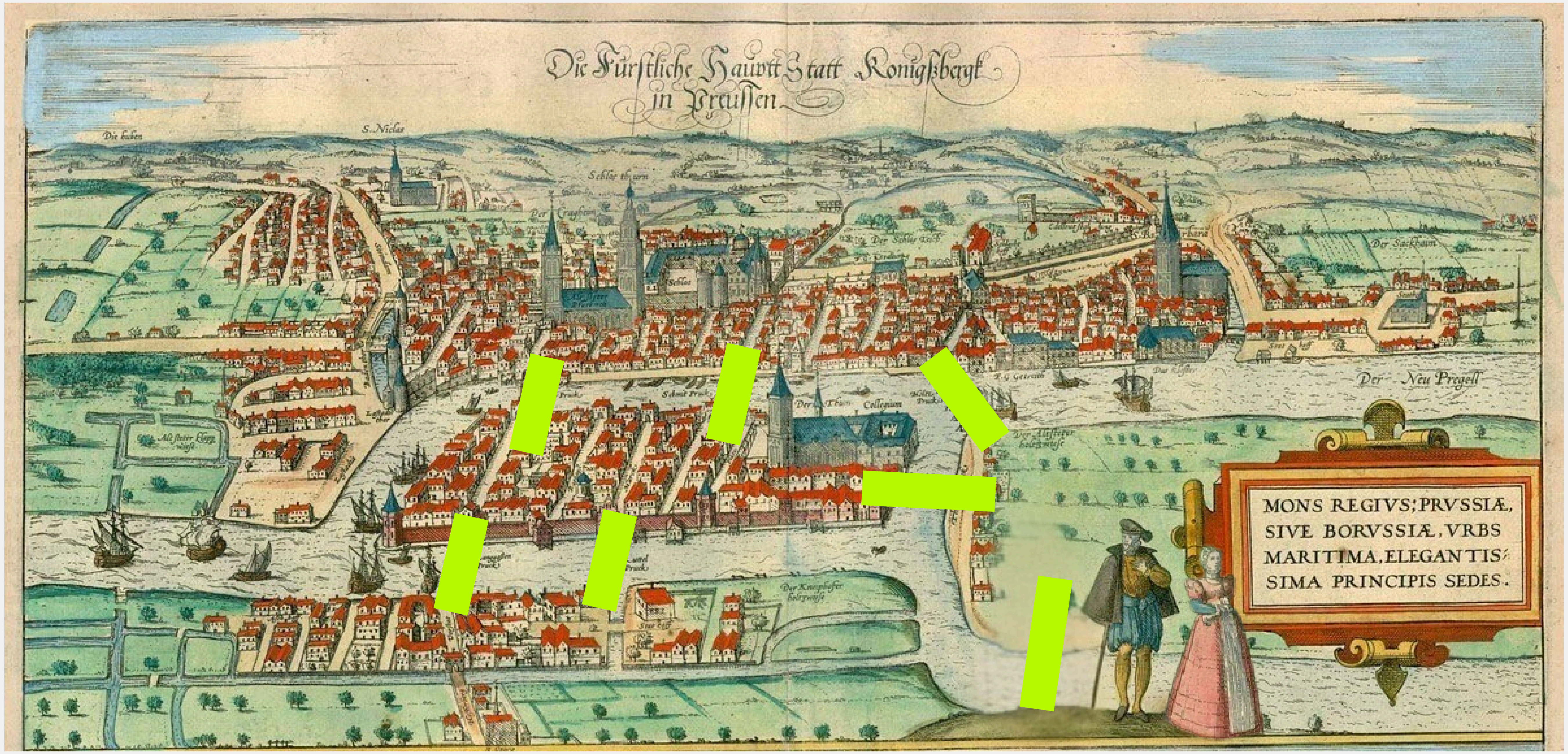
The seven bridges of Königsberg



Source: https://www.amusingplanet.com/2018/08/the-seven-bridges-of-konigsberg.html

THE PUZZLE THAT STARTED IT ALL

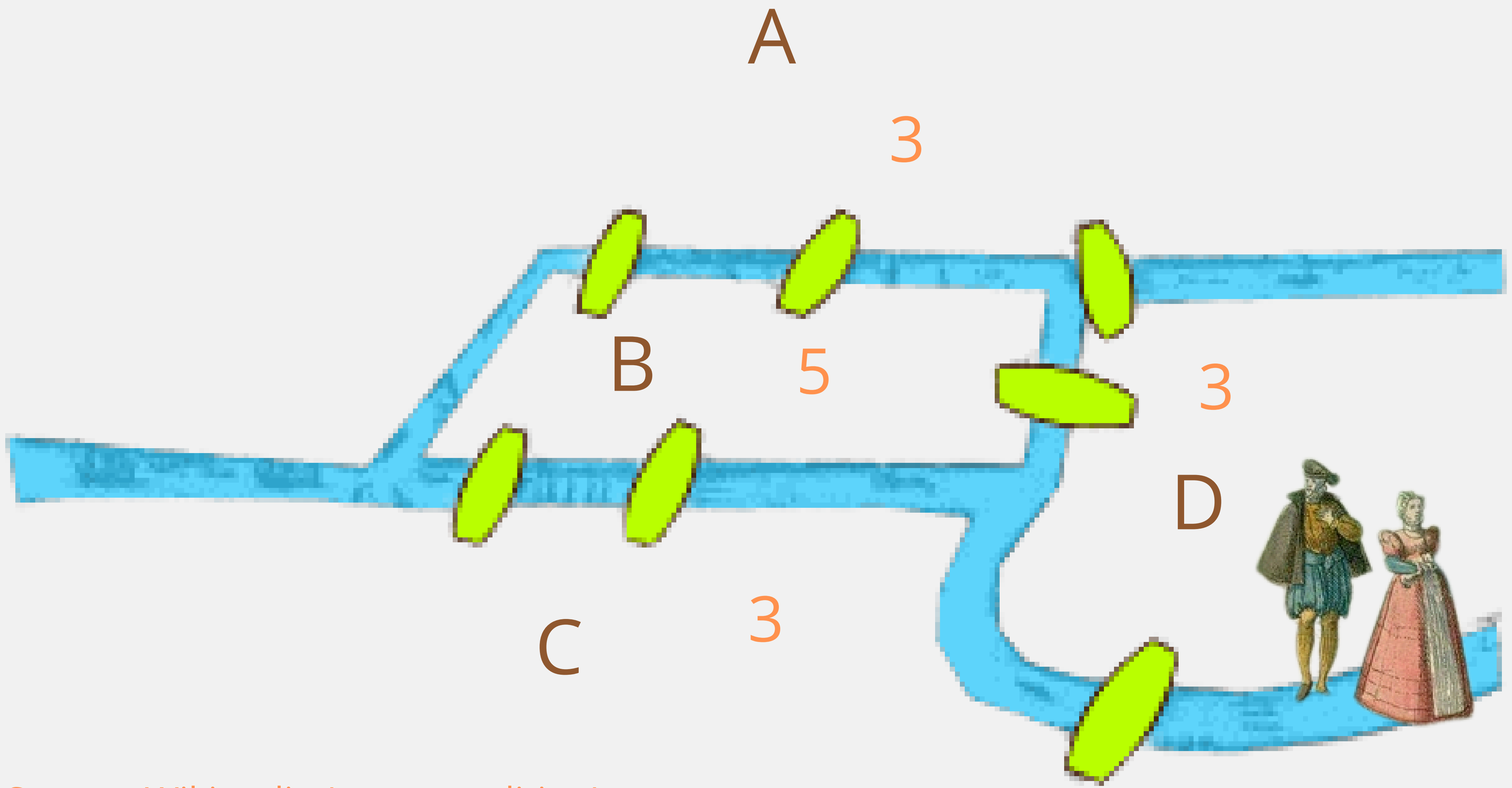
The seven bridges of Königsberg



Source: https://www.amusingplanet.com/2018/08/the-seven-bridges-of-konigsberg.html

THE PUZZLE THAT STARTED IT ALL

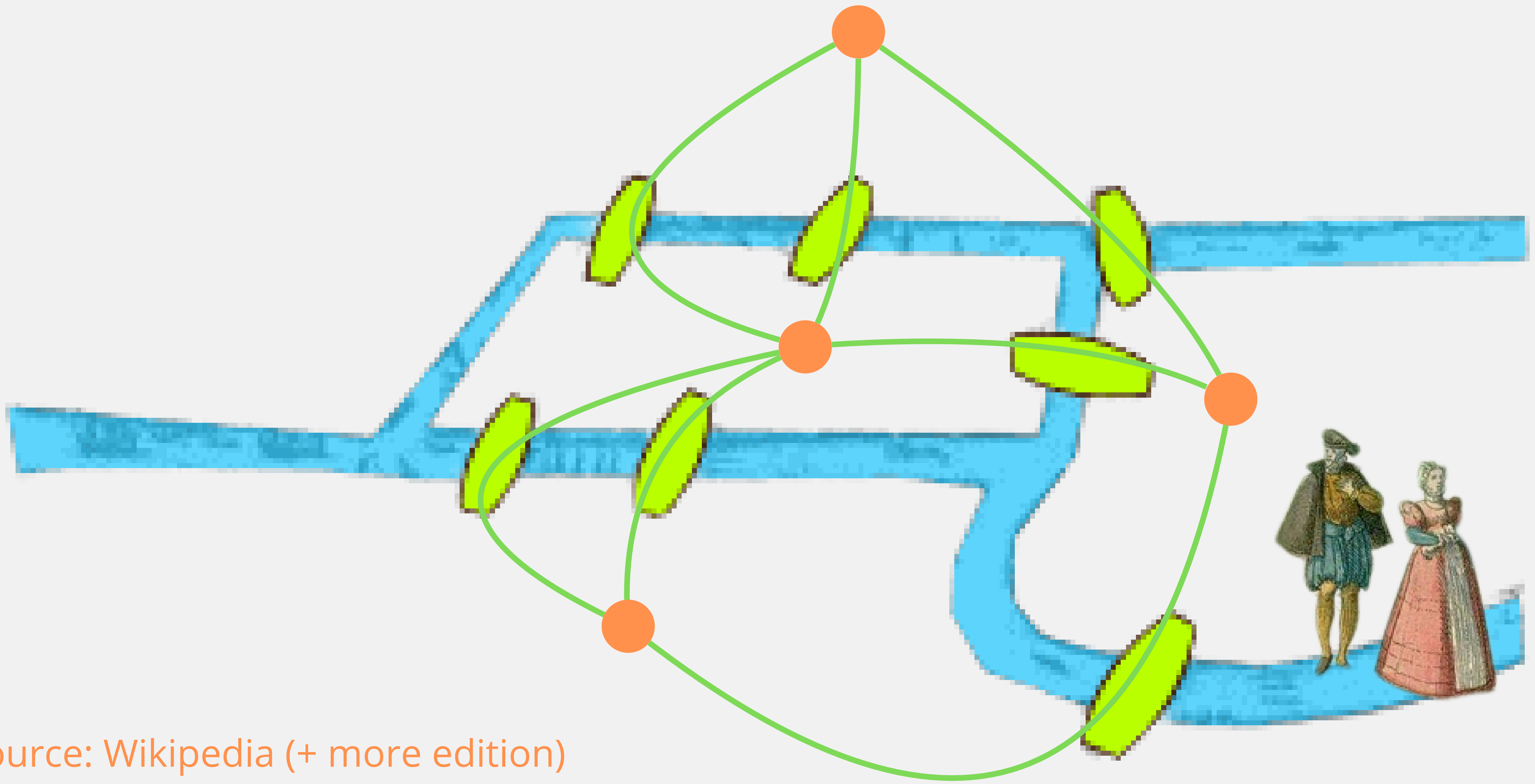
The seven bridges of Königsberg



Source: Wikipedia (+ some edition)

THE PUZZLE THAT STARTED IT ALL

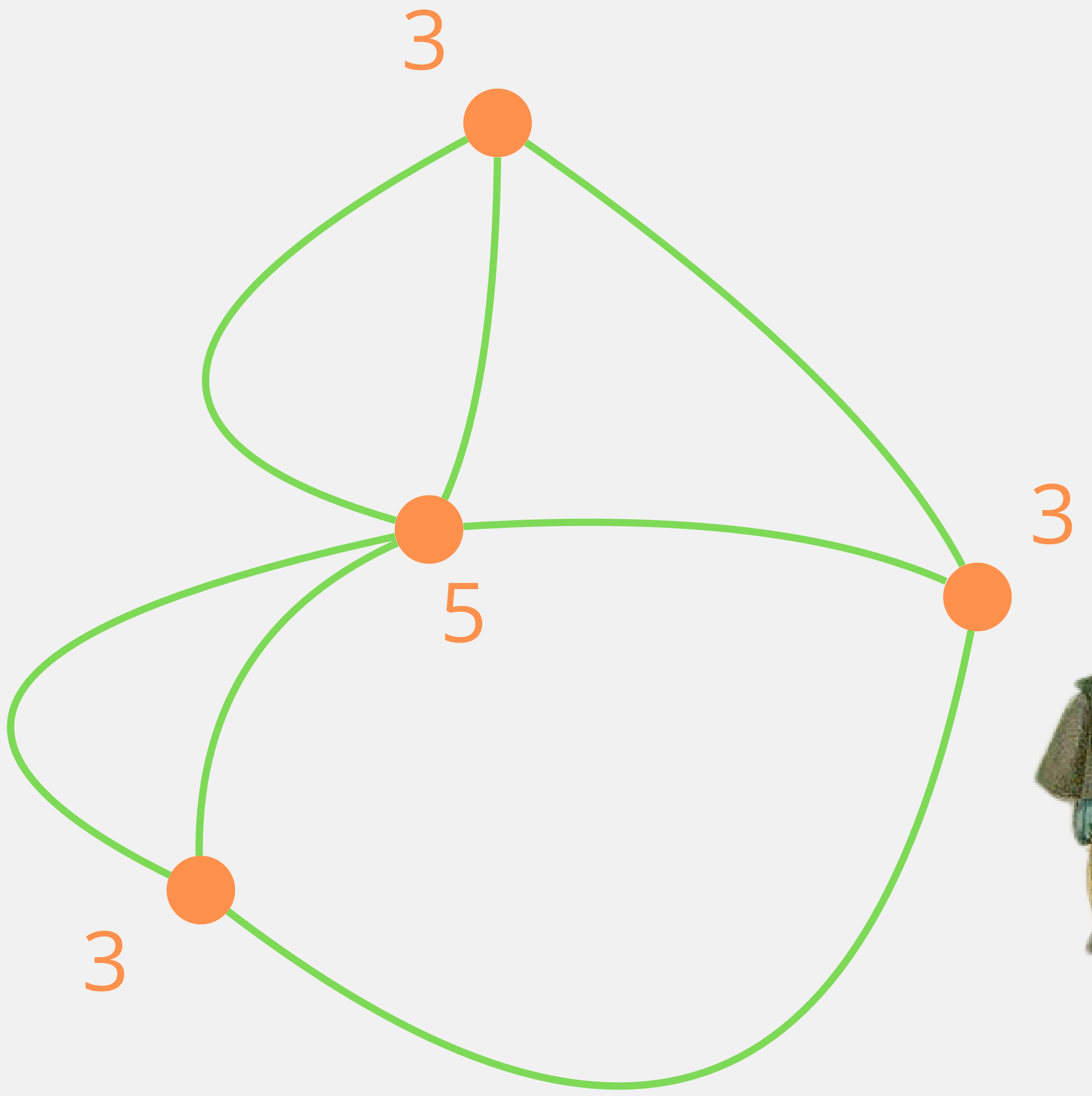
The seven bridges of Königsberg



Source: Wikipedia (+ more edition)

THE PUZZLE THAT STARTED IT ALL

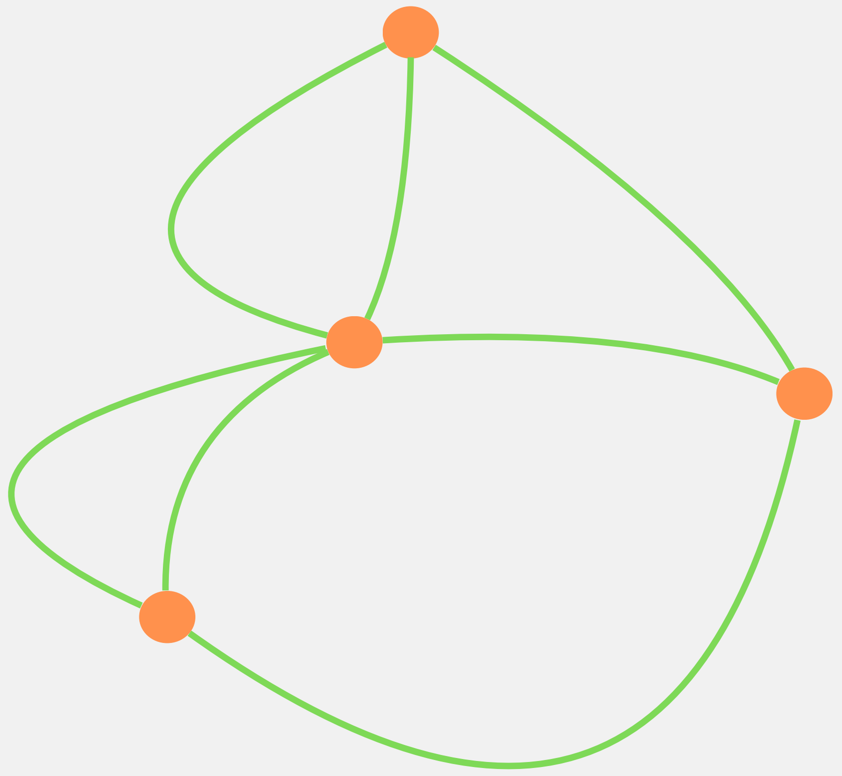
The seven bridges of Königsberg

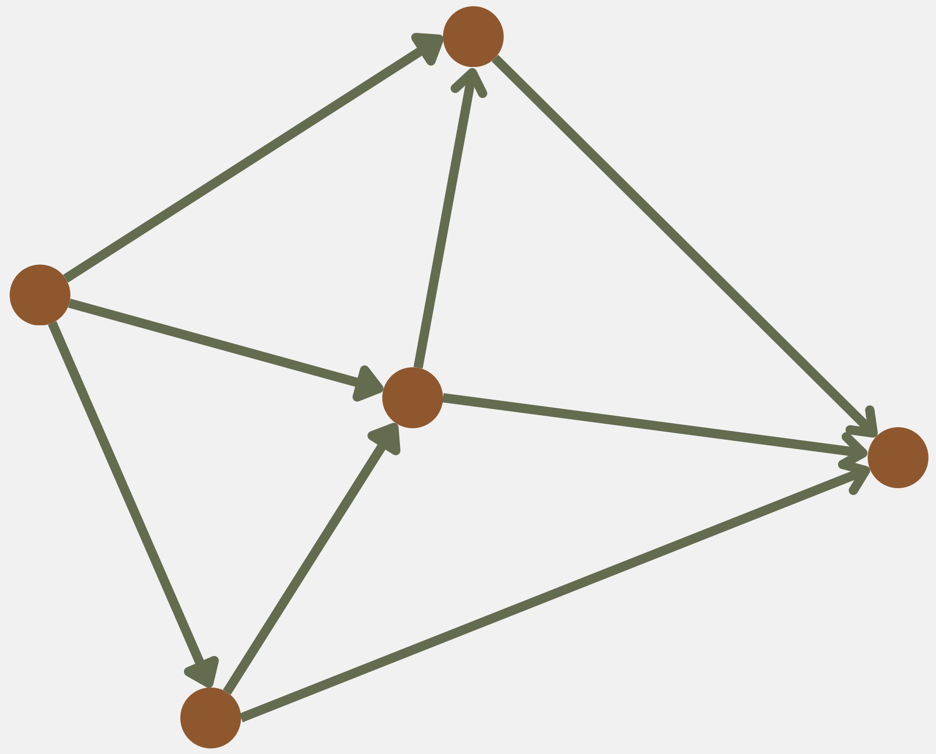


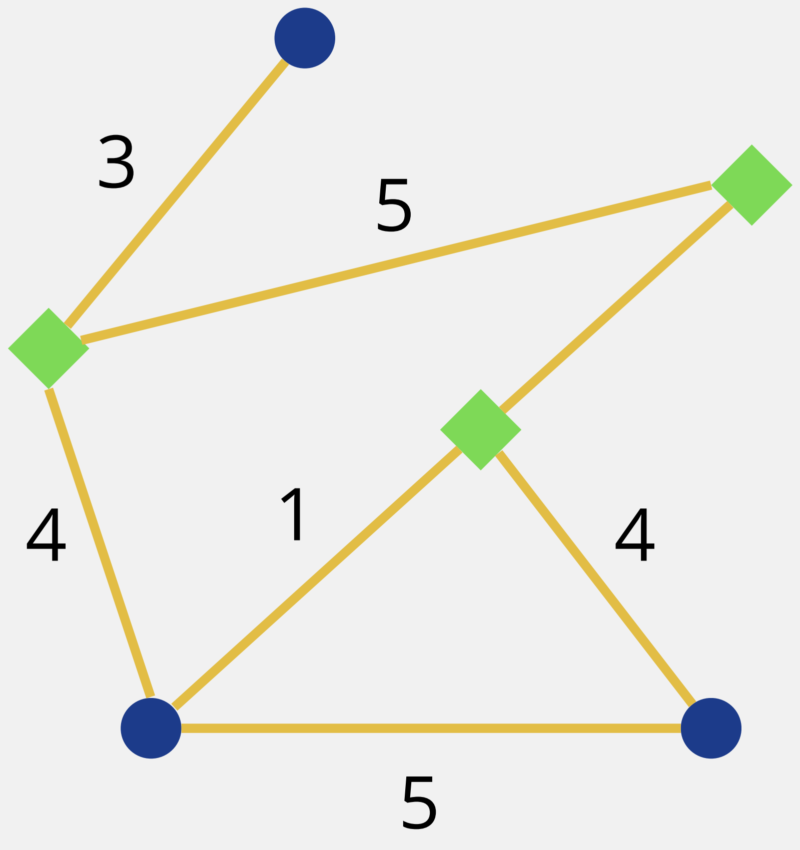
WHAT IS A GRAPH?

WHAT IS A GRAPH?

A graph is a set of **vertices** (*dots, points, nodes*) and a set of **edges** (*connections*)







COMPUTATIONAL PROBLEMS

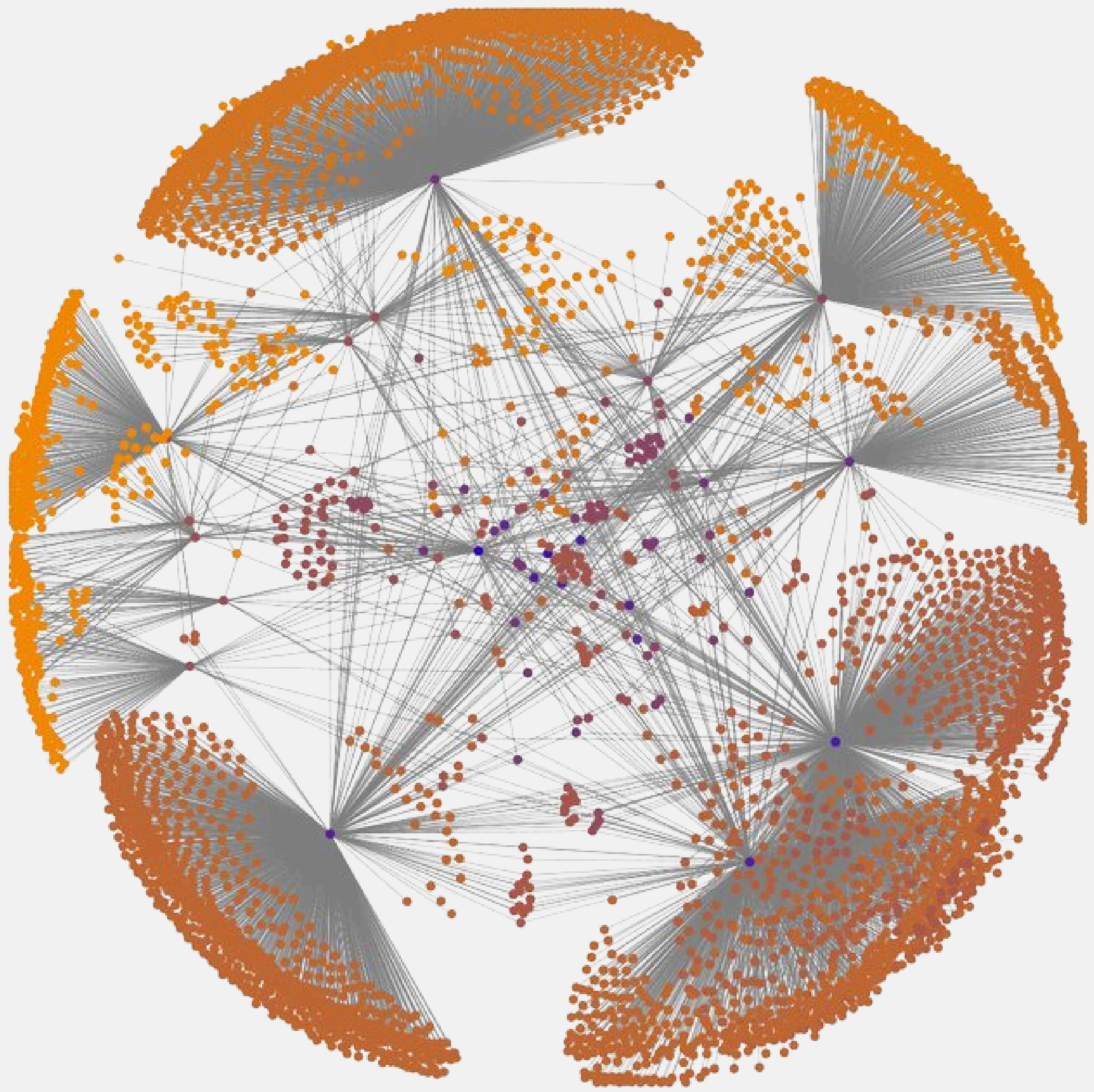
COMPUTATIONAL PROBLEMS

PageRank

How are webpages ordered when users perform a Google search?

COMPUTATIONAL PROBLEMS

PageRank

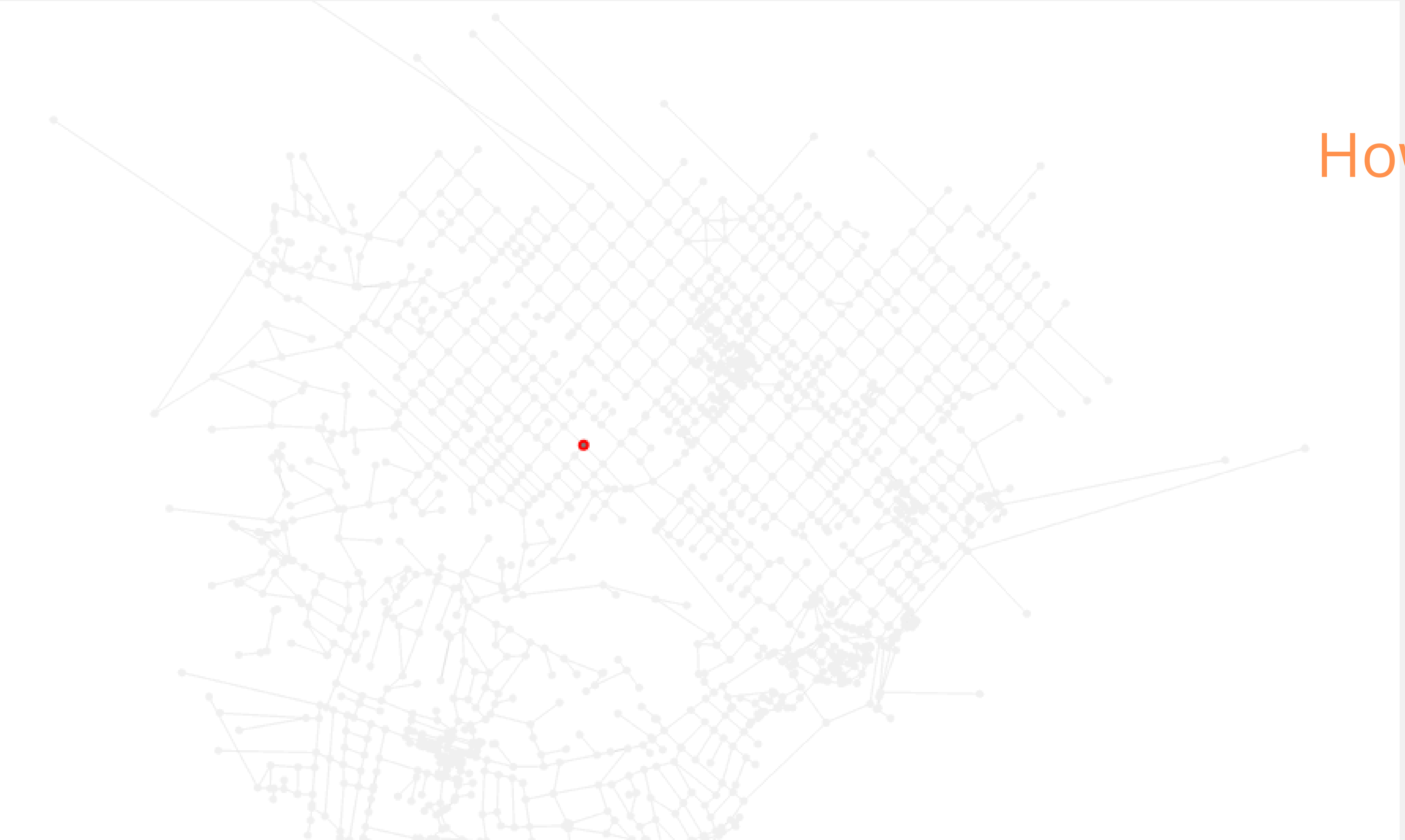


How are webpages ordered when users perform a Google search?

Source: Motif Simplification: Improving Network Visualization Readability with Fan and Parallel Glyphs (Dunne, Shneiderman) 2013

COMPUTATIONAL PROBLEMS

Routing Problems

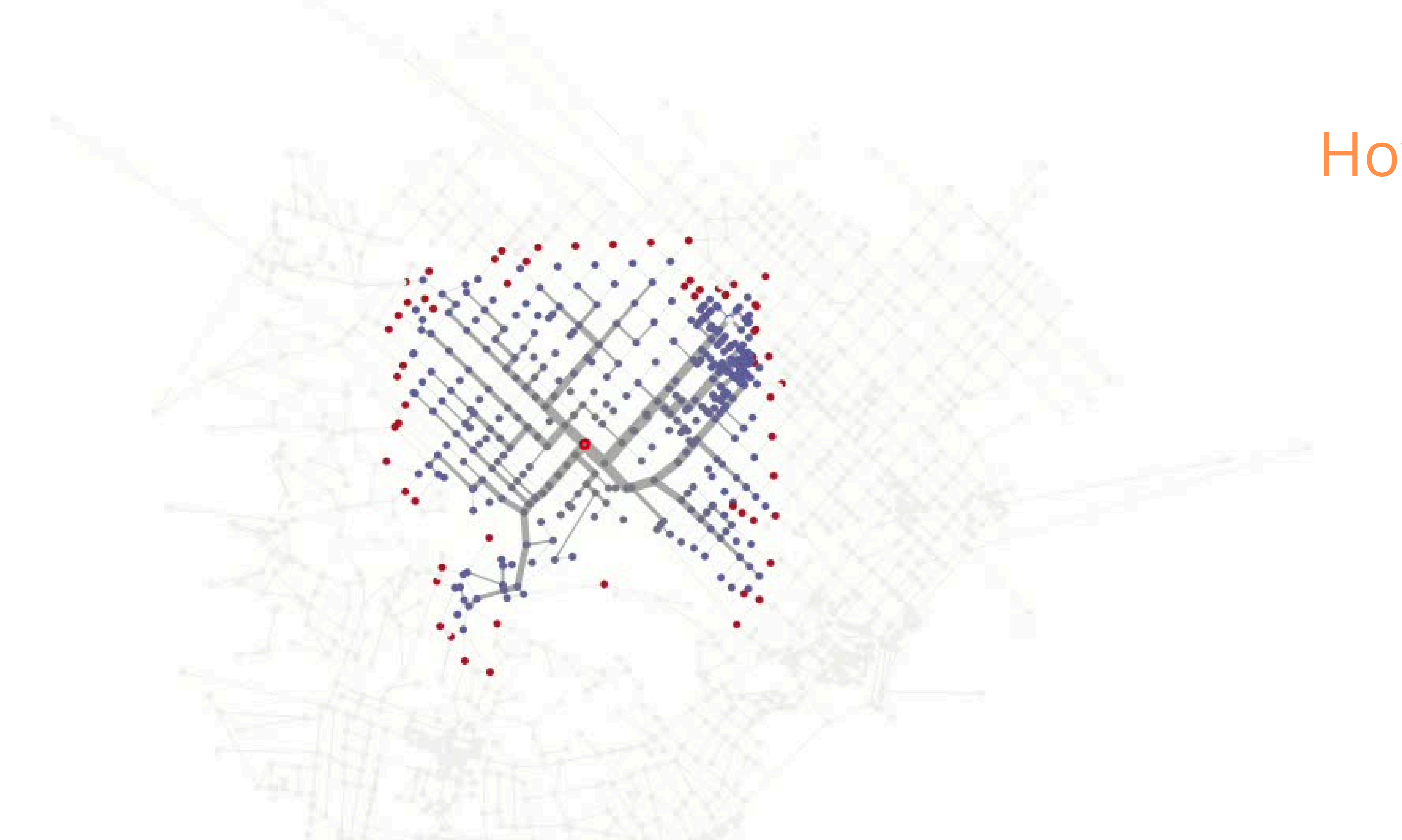


Source: projects.indicatrix.org/dijkstra.js/

How does Google maps suggest the shortest route from A to B?

COMPUTATIONAL PROBLEMS

Routing Problems

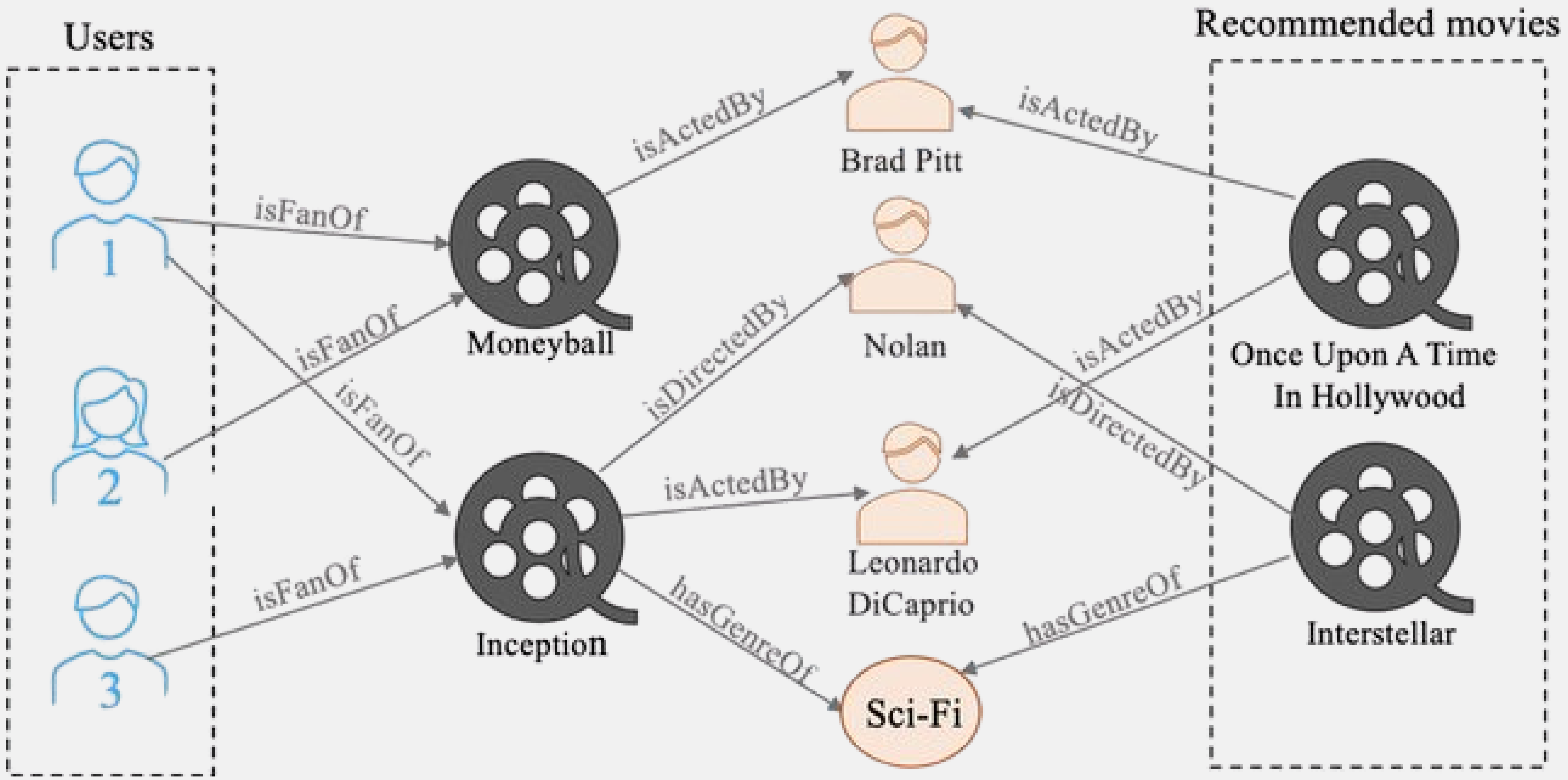


Source: projects.indicatrix.org/dijkstra.js/

How does Google maps suggest the shortest route from A to B?

COMPUTATIONAL PROBLEMS

Recommendation systems

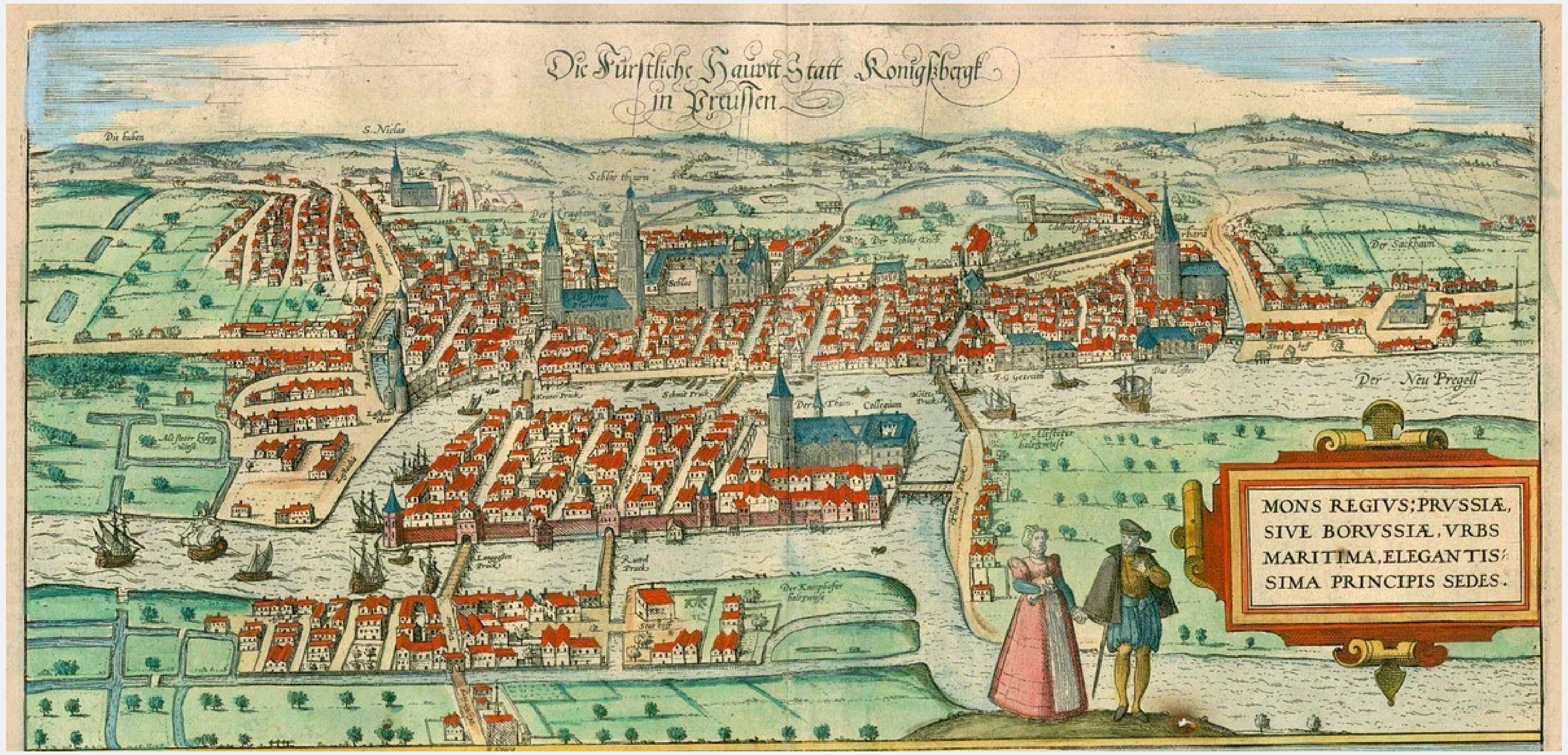


How does Netflix recommend movies?

Source: Knowledge Graphs: Opportunities and Challenges (Peng, Xia, Naseriparsa, Osborne) Artificial Intelligence Review, 2023

COMPUTATIONAL PROBLEMS

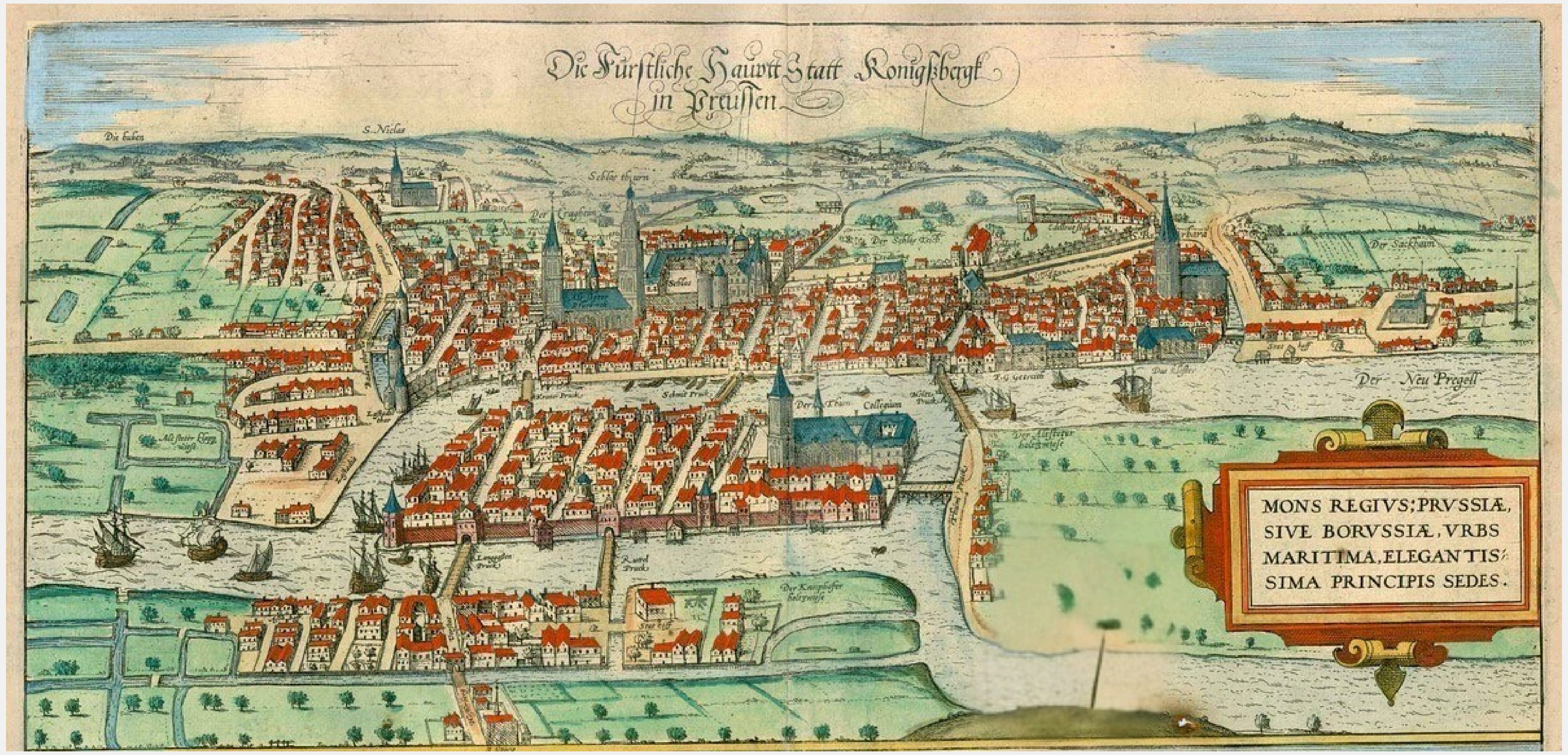
Pattern recognition



How do computers recognize figures (patterns)?

COMPUTATIONAL PROBLEMS

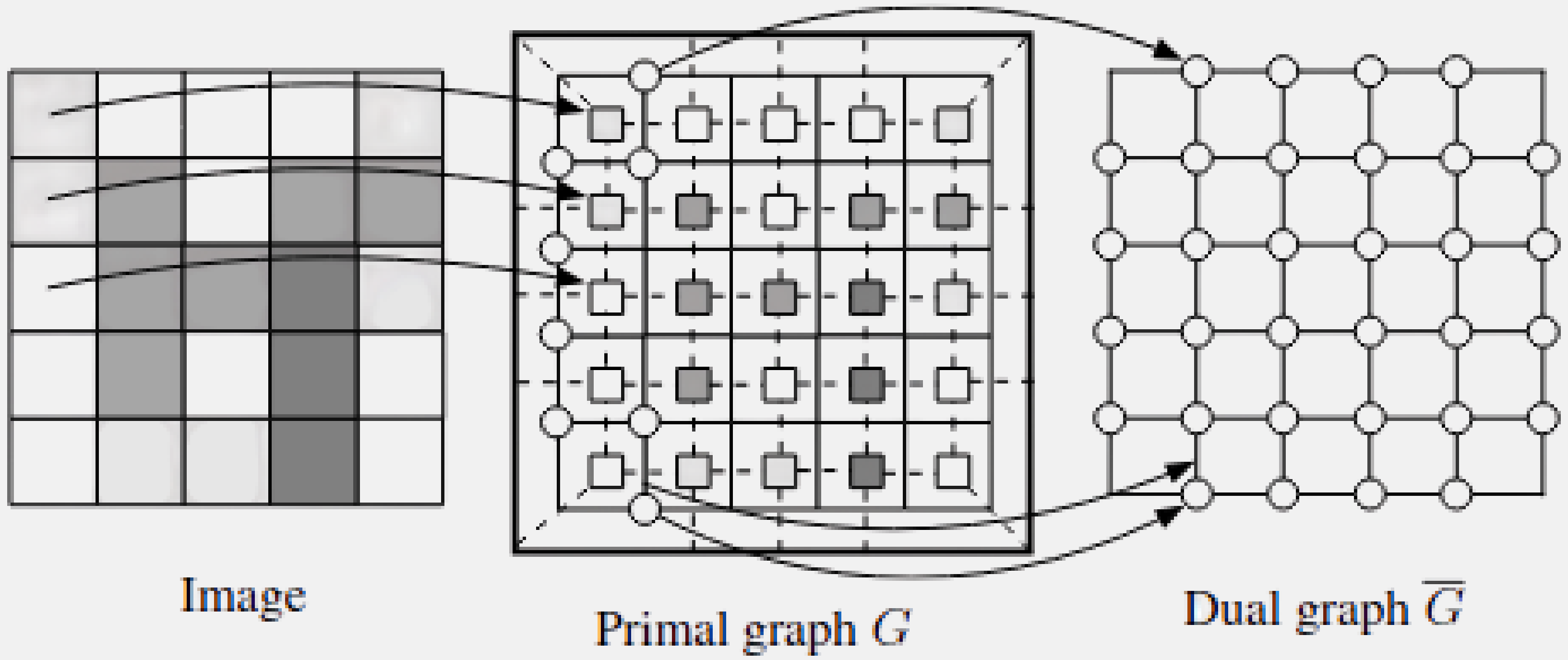
Pattern recognition



How do computers recognize figures (patterns)?

COMPUTATIONAL PROBLEMS

Pattern recognition

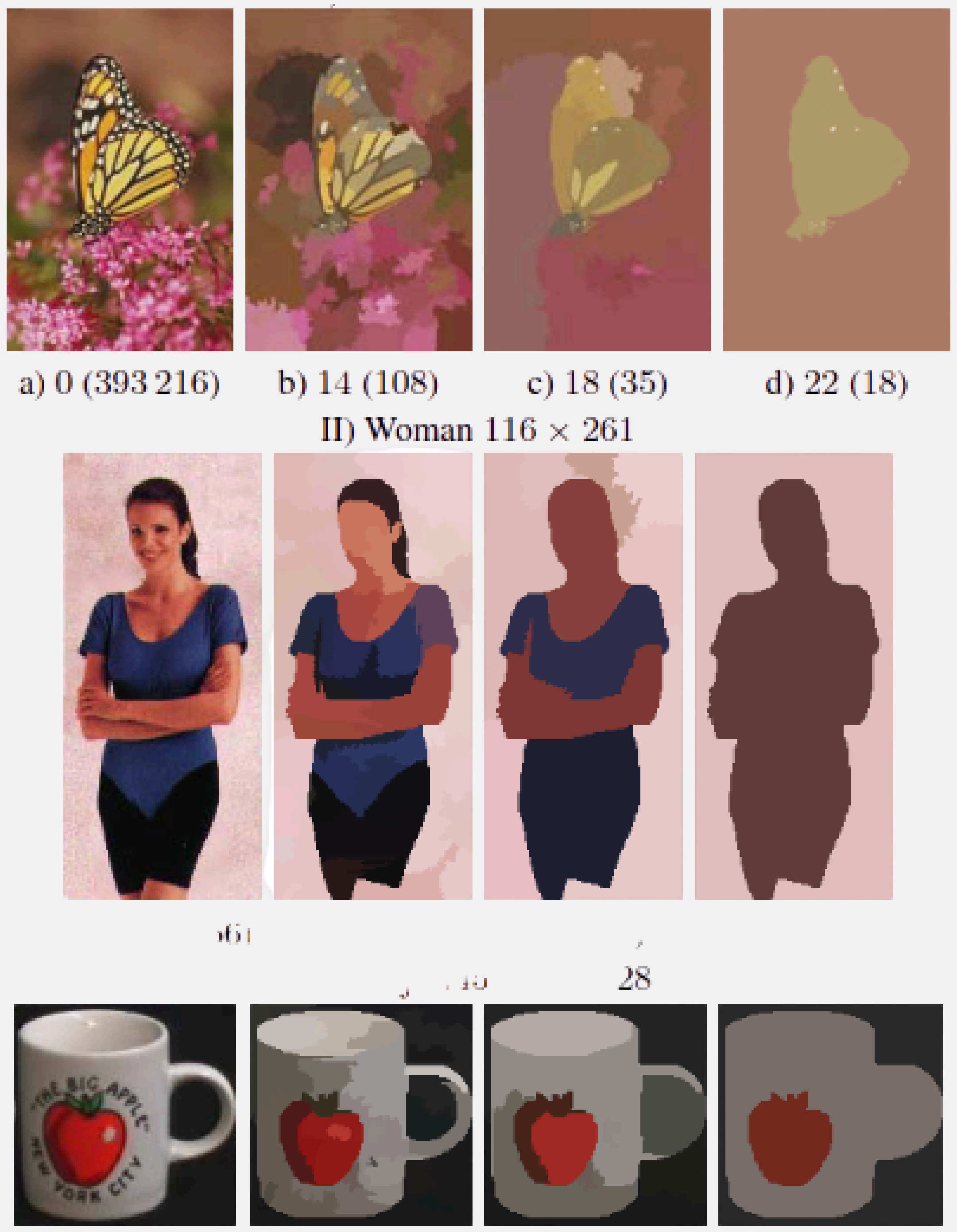


How do computers recognize figures (patterns)?

Source:Multiresolution Image Segmentations in Graph Pyramids (Kropatsch, Haxhimusa, Ion) 2007

COMPUTATIONAL PROBLEMS

Pattern recognition

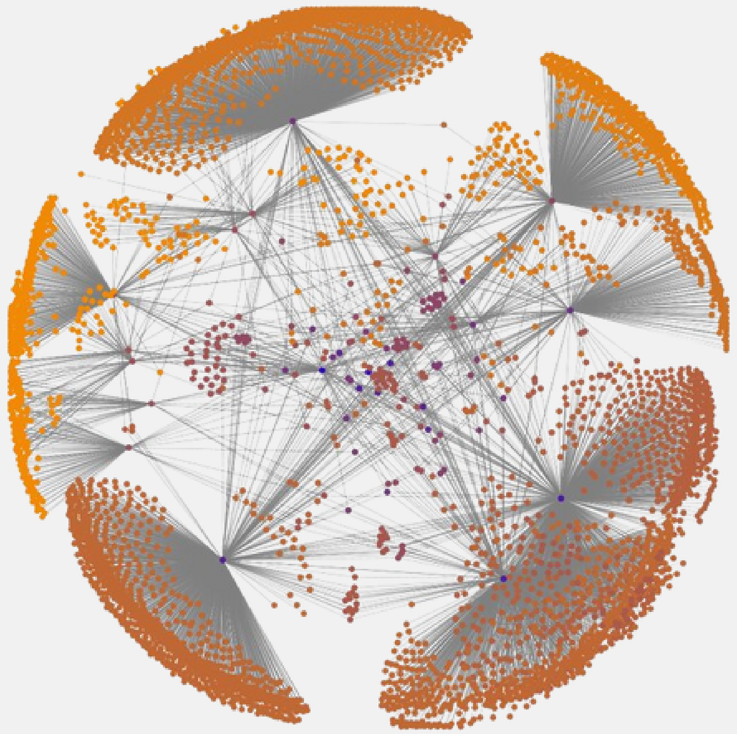


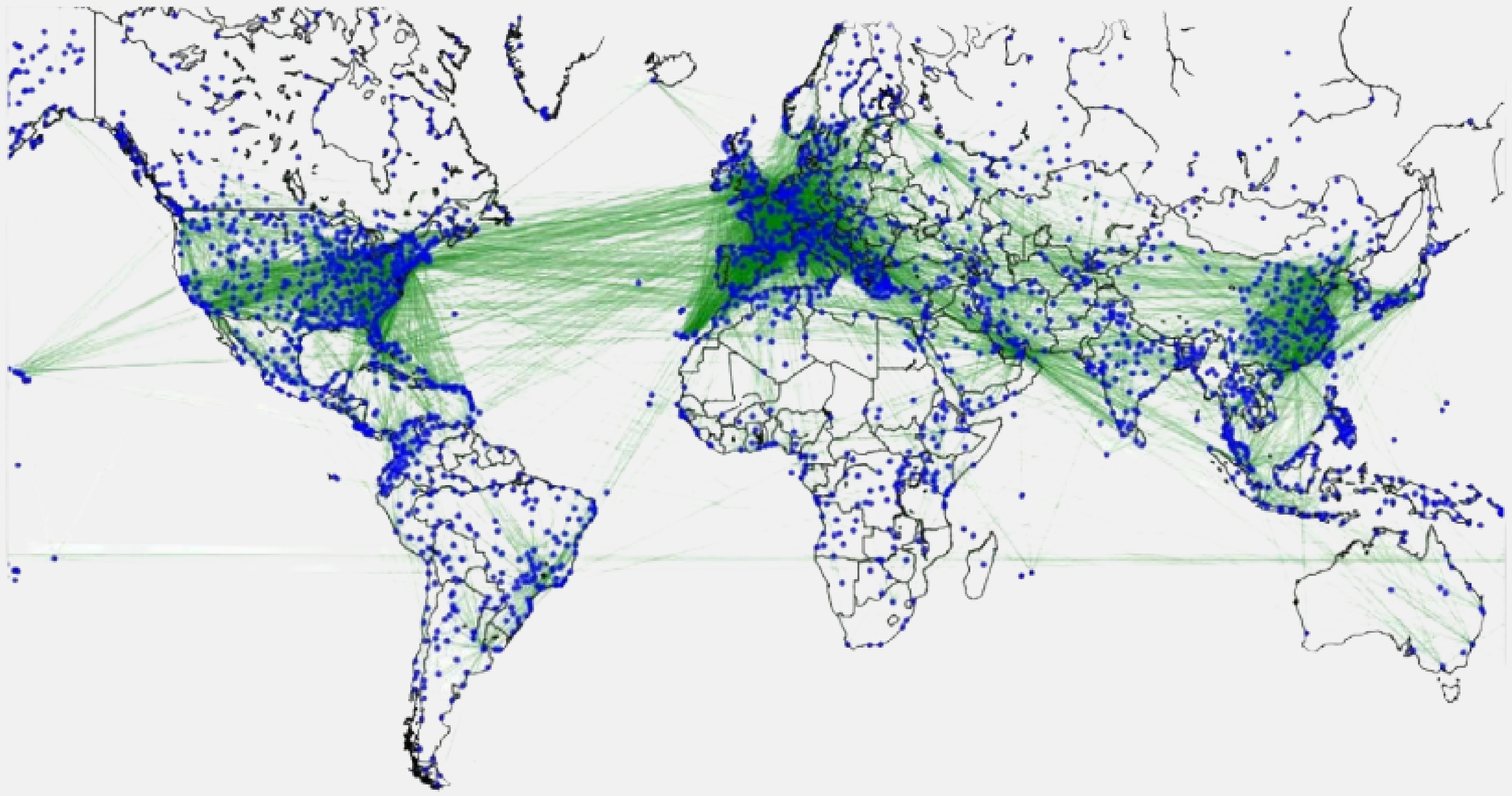
How do computers recognize figures (patterns)?

Source:Multiresolution Image Segmentations in Graph Pyramids (Kropatsch, Haxhimusa, Ion) 2007

COMPUTATIONAL PROBLEMS

Efficiency matters



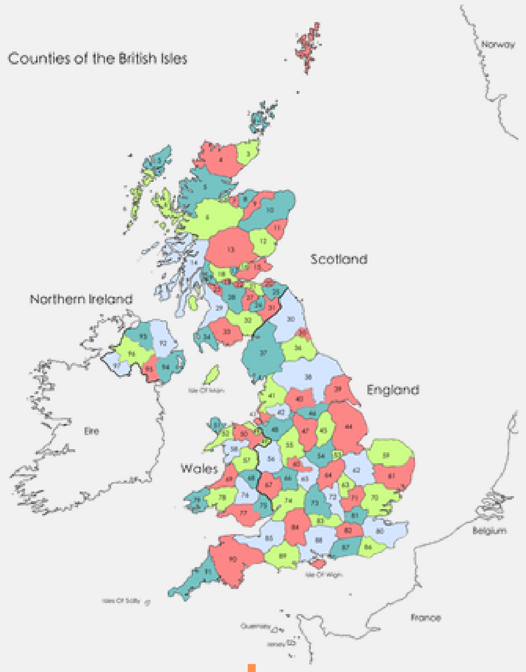


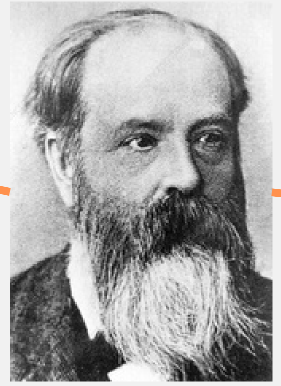
COLOURING PROBLEMS

The Four Colour Theorem

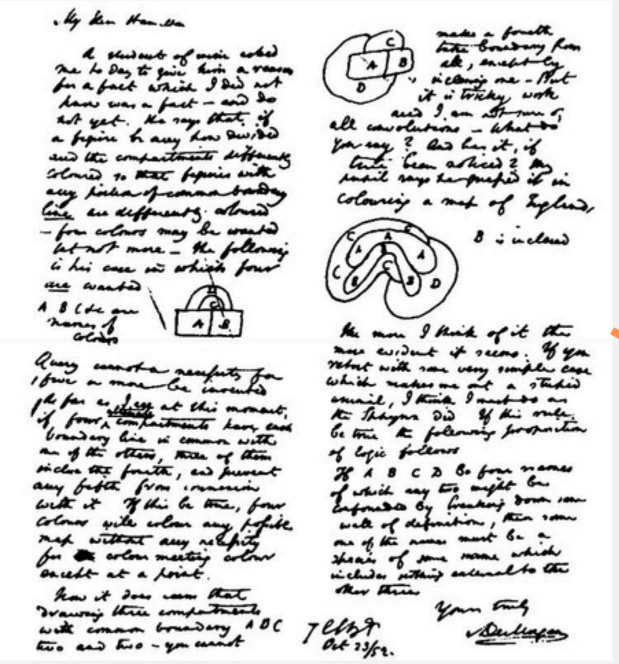
COLOURING PROBLEMS

The Four Colour Conjecture (1852)





Frederick Guthrie



My dear Hamilton

A student of mine asked me to day to give him a reason for a fact which I did not know was a fact — and do not yet. He says that if a figure be any how divided and the compartments differently coloured so that figures with any portion of common boundary line are differently coloured — four colours may be wanted but not more — the following is his case in which four are wanted

A B C &c are names of colours

Query cannot a necessity for five or more be invented? As far as I see at this moment, if four ultimate compartments have each boundary line in common with one of the others, three of them inclose the fourth, and prevent any fifth from connexion with it. If this be true, four colours will colour any possible map without any necessity for colour meeting colour except at a point.

Now it does seem that drawing three compartments with common boundary ABC two and two — you cannot make a fourth take boundary from all, except by inclosing one — But it is tricky work and I am not sure of all convolutions — What do you say? And has it, if true been noticed? My pupil says he guessed it in colouring a map of England.

B is inclosed

The more I think of it the more evident it seems. If you retort with some very simple case which makes me out a stupid animal, I think I must do as the Sphynx did. If this rule be true the following proposition of logic follows

If A B C D be four names of which any two might be confounded by breaking down some wall of definition, then some one of the names must be a species of some name which includes nothing external to the other three

Yours truly
A De Morgan

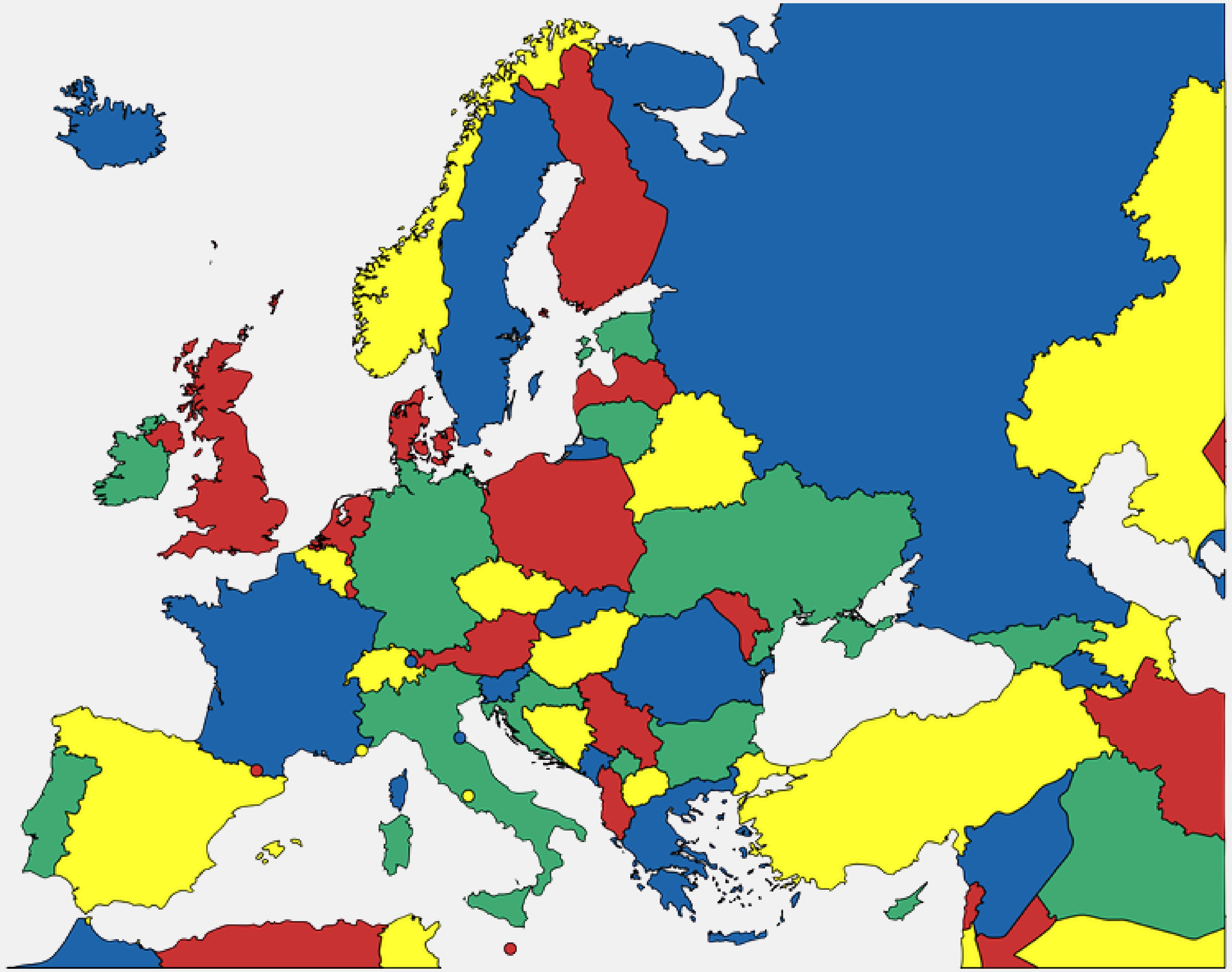
7 Camden St
Oct 23/52.

Francis Guthrie

Augustus de Morgan

COLOURING PROBLEMS

The Four Colour Conjecture (1852)



Source: Reddit

COLOURING PROBLEMS

The Four Colour Conjecture (1852)

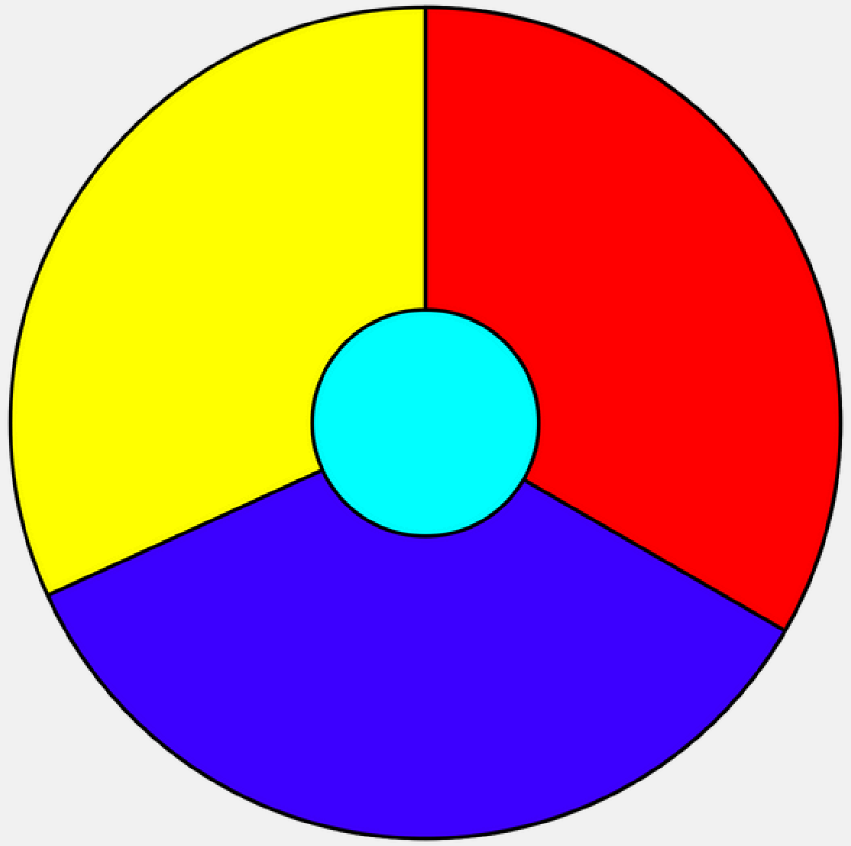
Source: Reddit

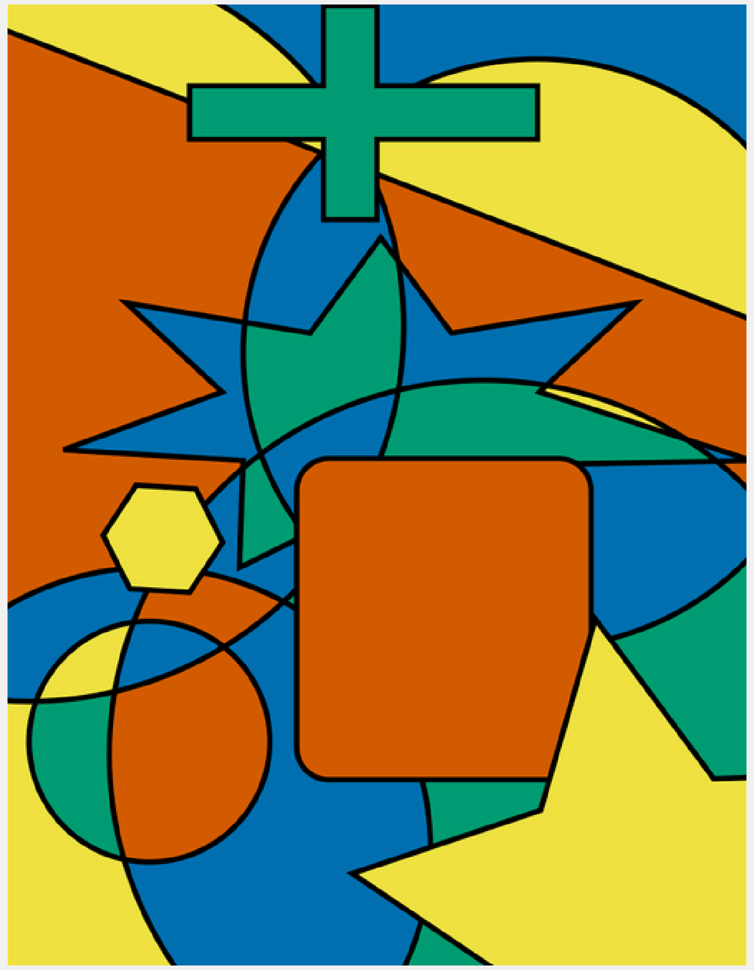
COLOURING PROBLEMS

The Four Colour Conjecture (1852)

COLOURING PROBLEMS

The Four Colour Conjecture (1852)





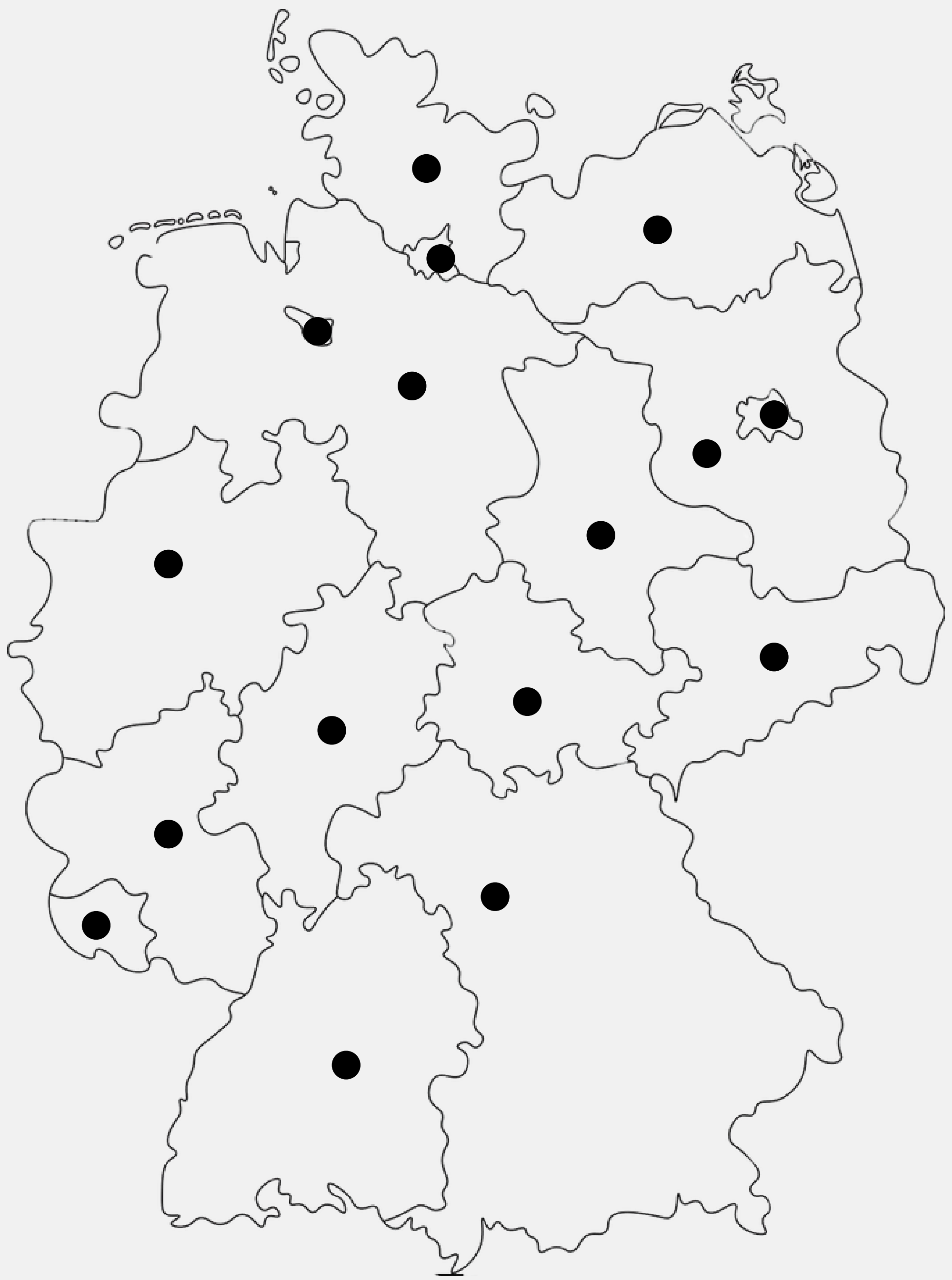
Source: Wikipedia

COLOURING PROBLEMS

The Four Colour Conjecture (1852)

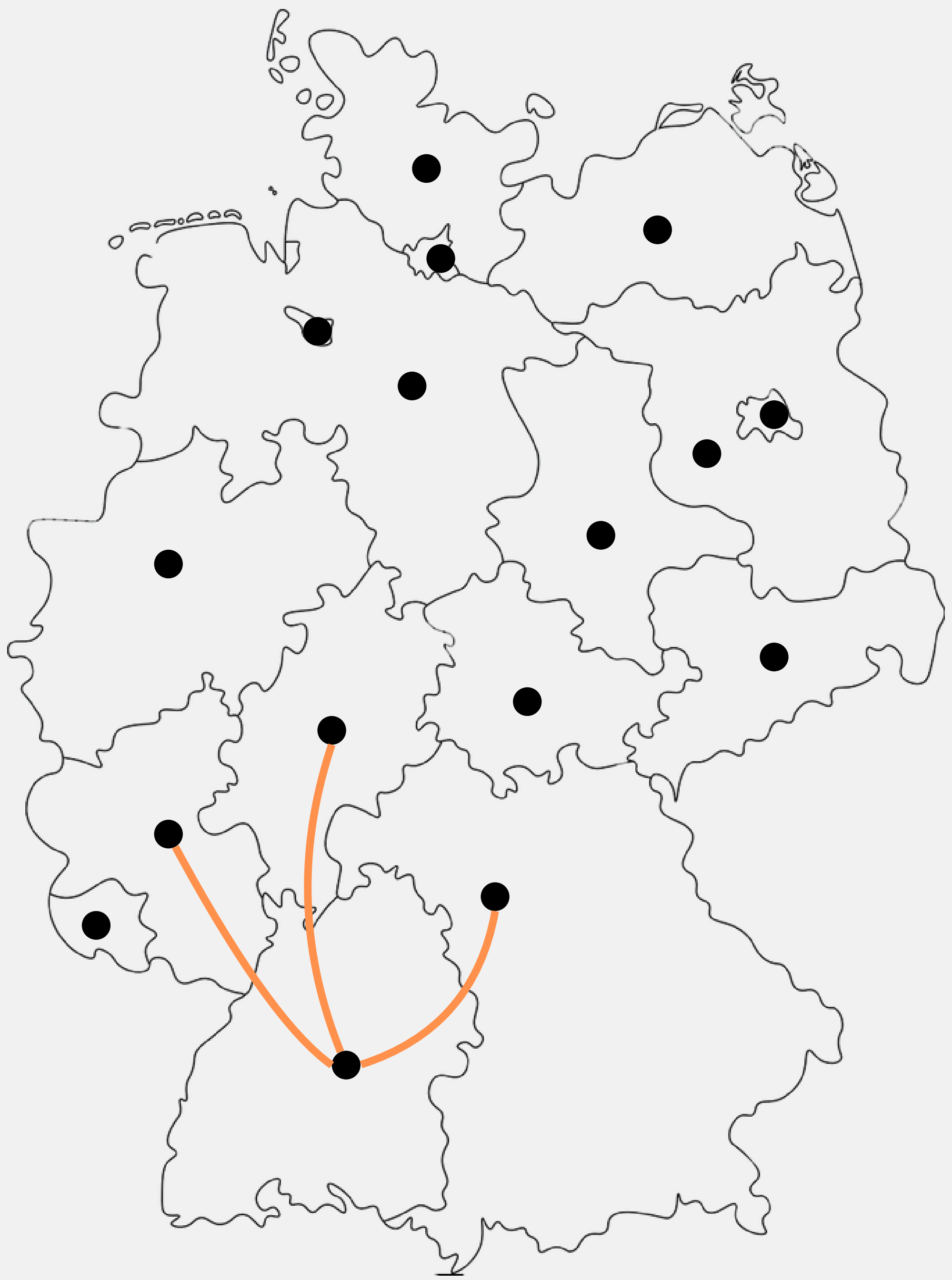
COLOURING PROBLEMS

The Four Colour Conjecture (1852)



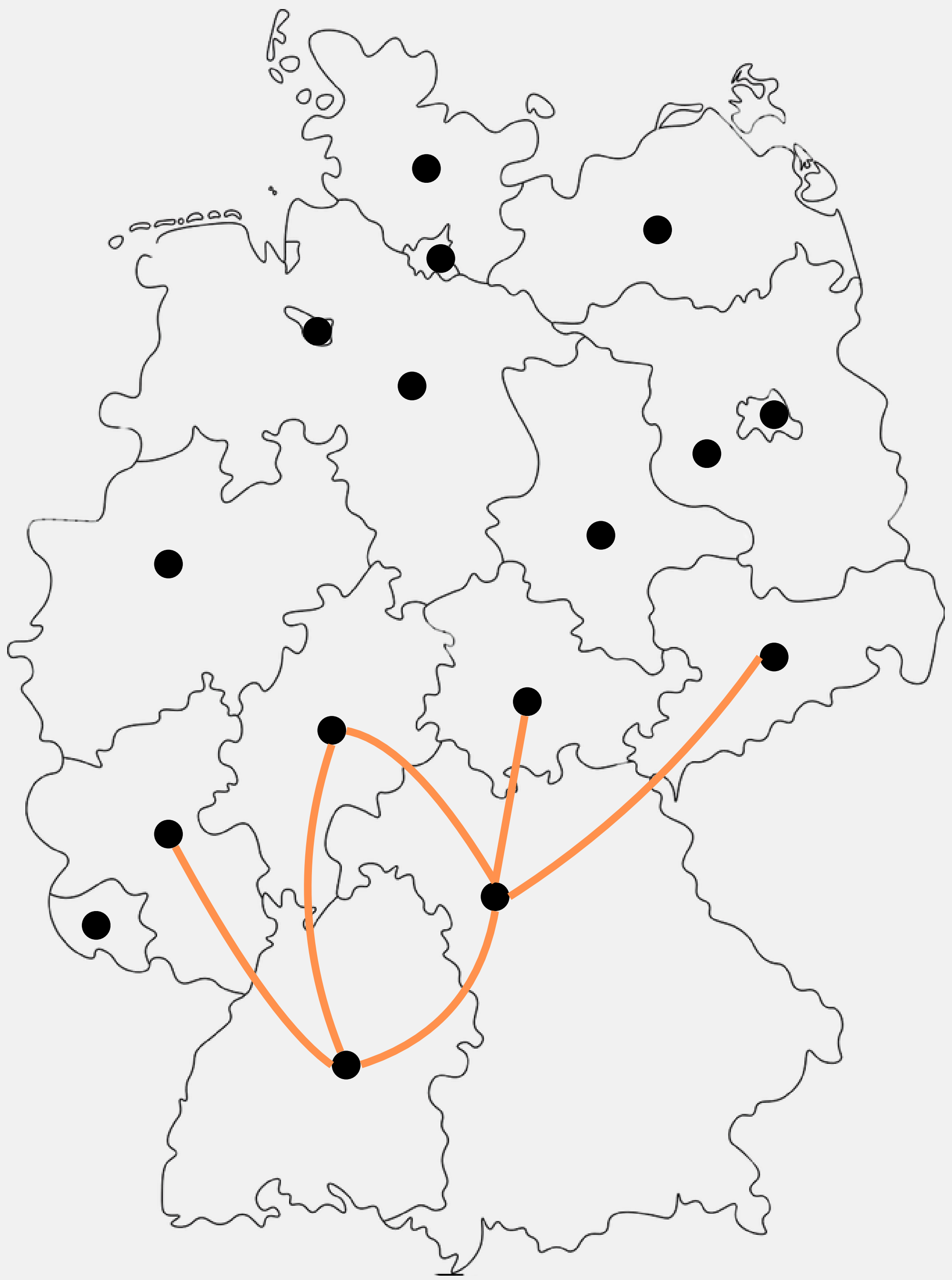
COLOURING PROBLEMS

The Four Colour Conjecture (1852)



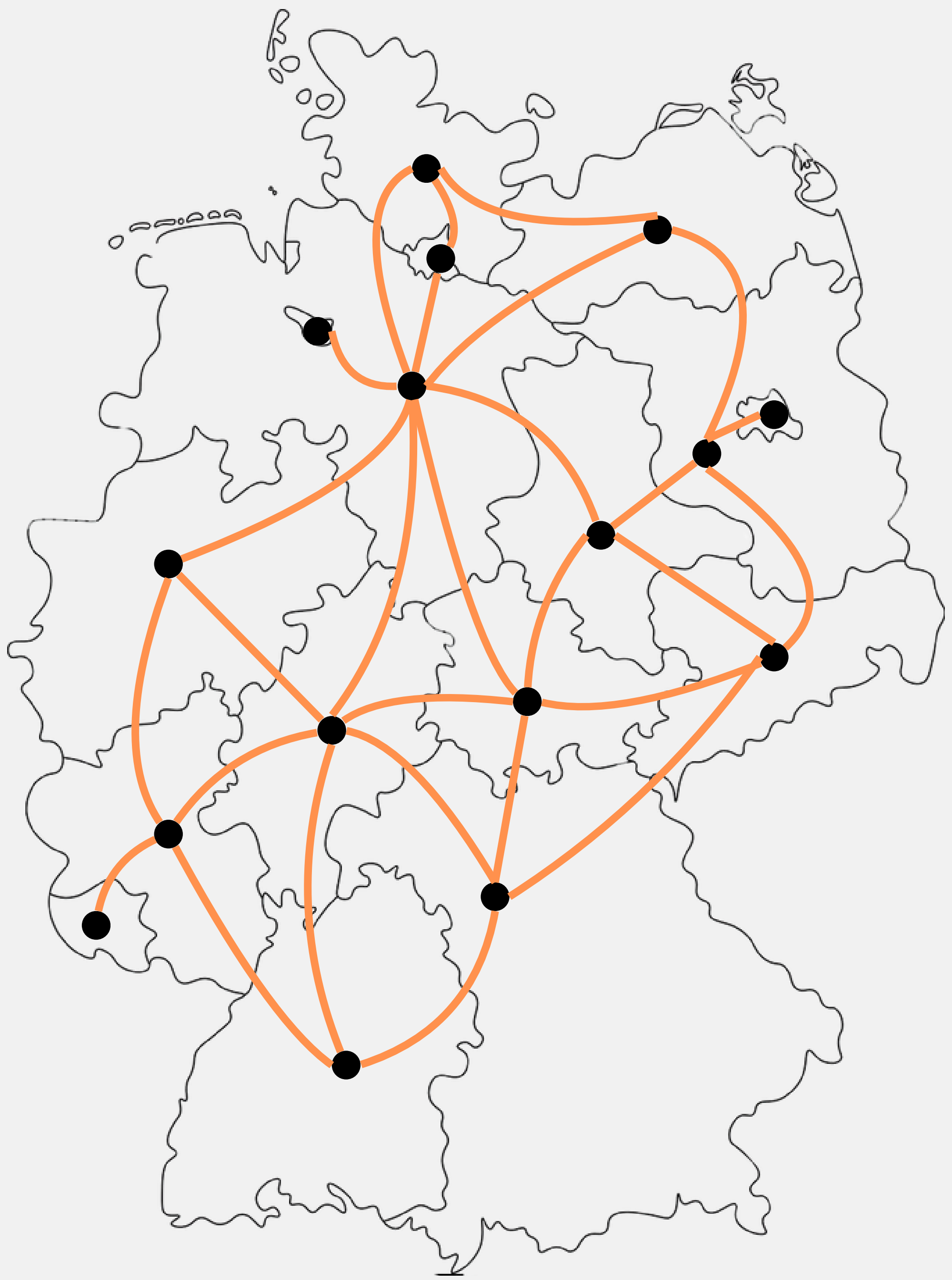
COLOURING PROBLEMS

The Four Colour Conjecture (1852)



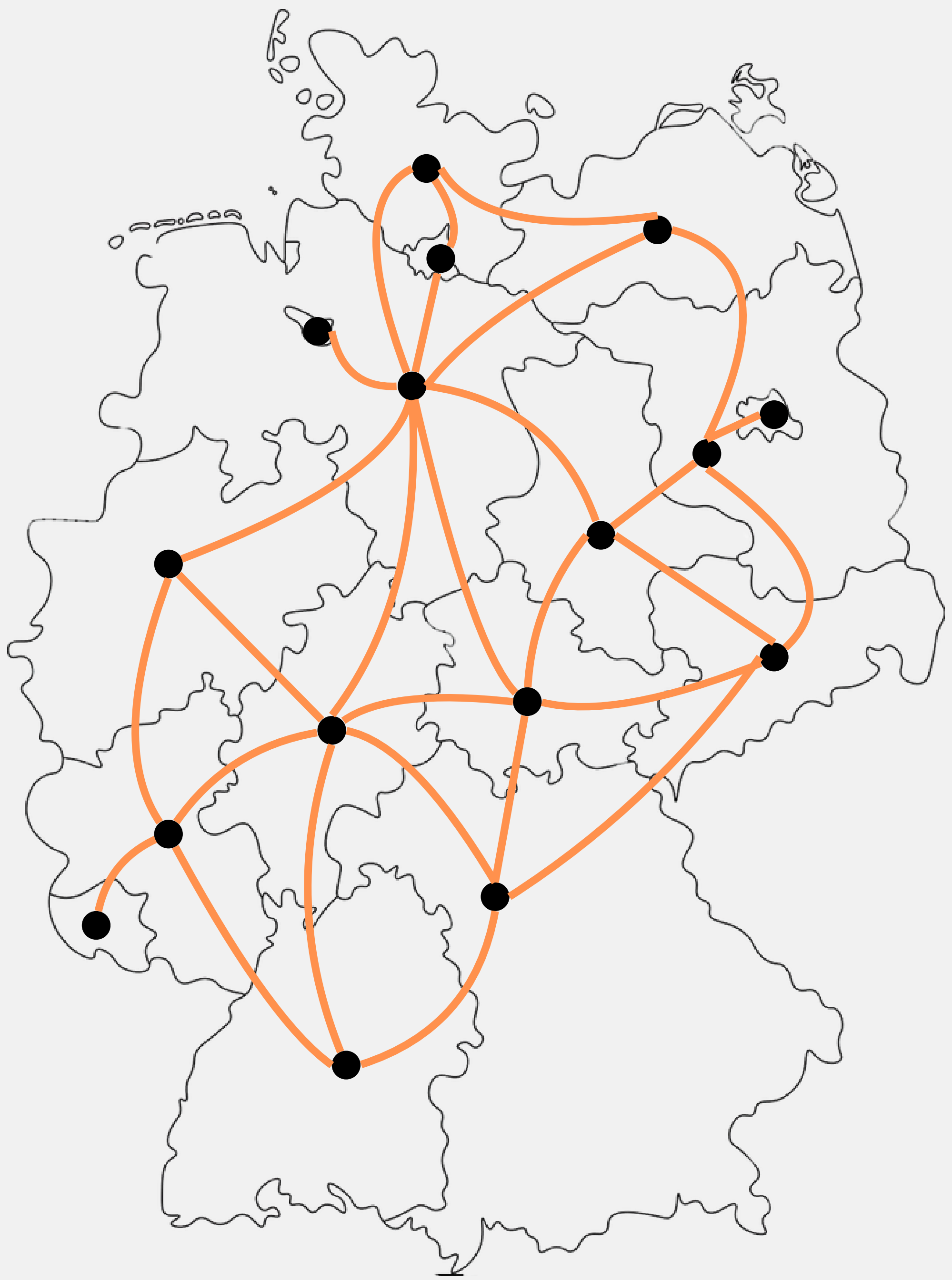
COLOURING PROBLEMS

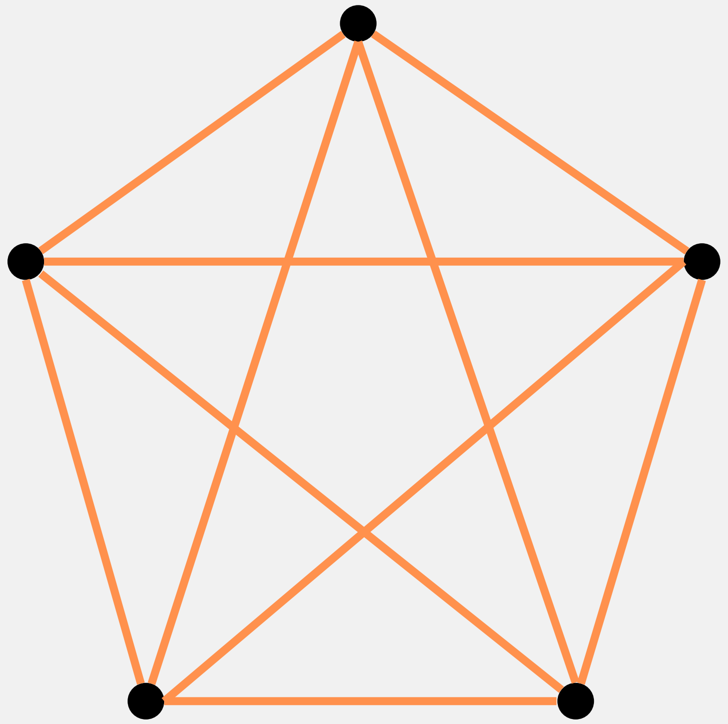
The Four Colour Conjecture (1852)



COLOURING PROBLEMS

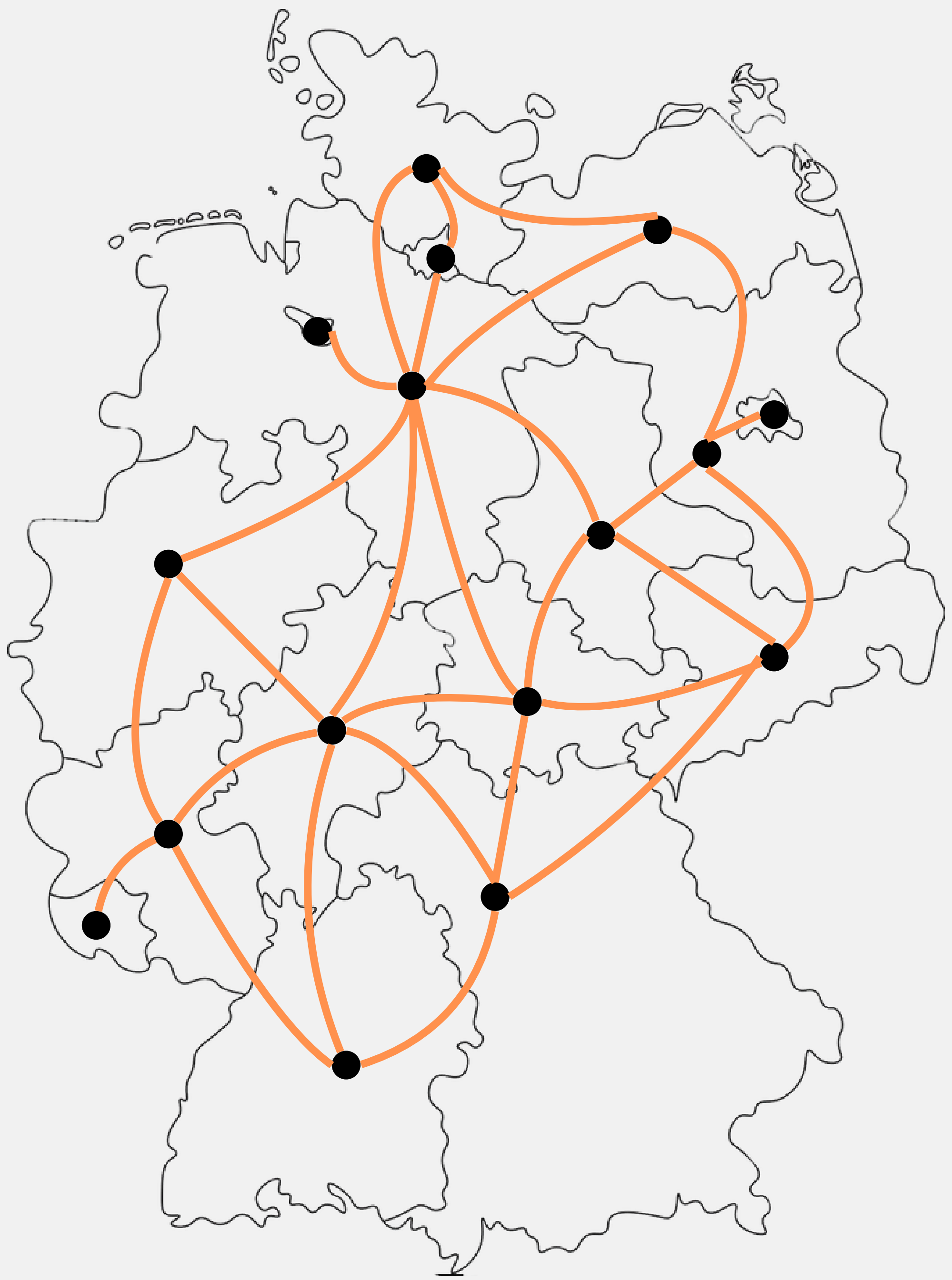
The Four Colour Conjecture (1852)

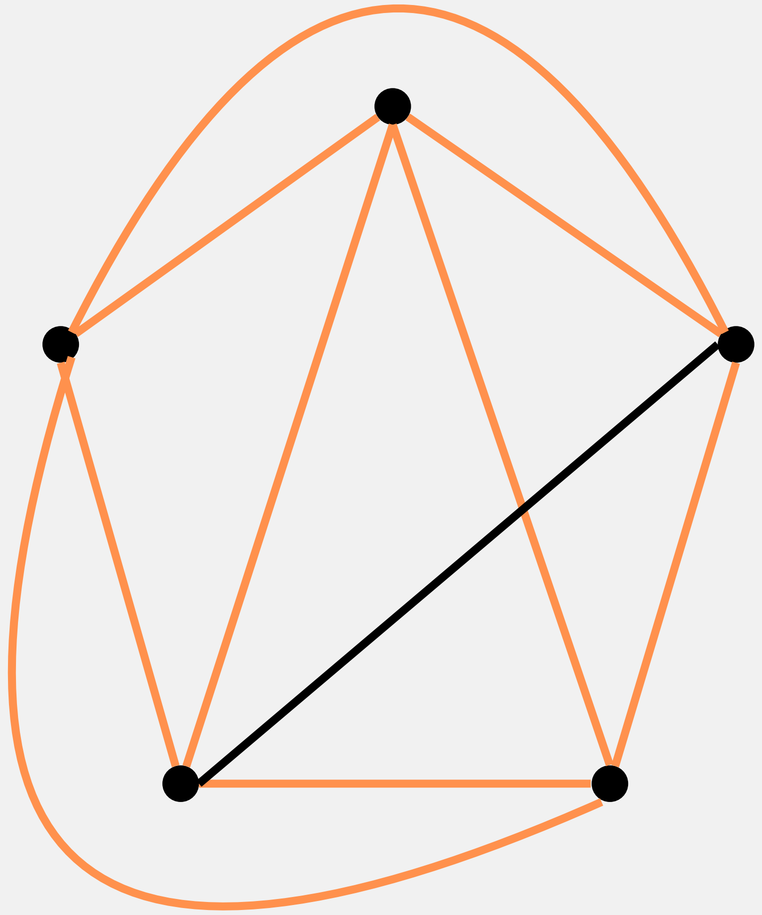




COLOURING PROBLEMS

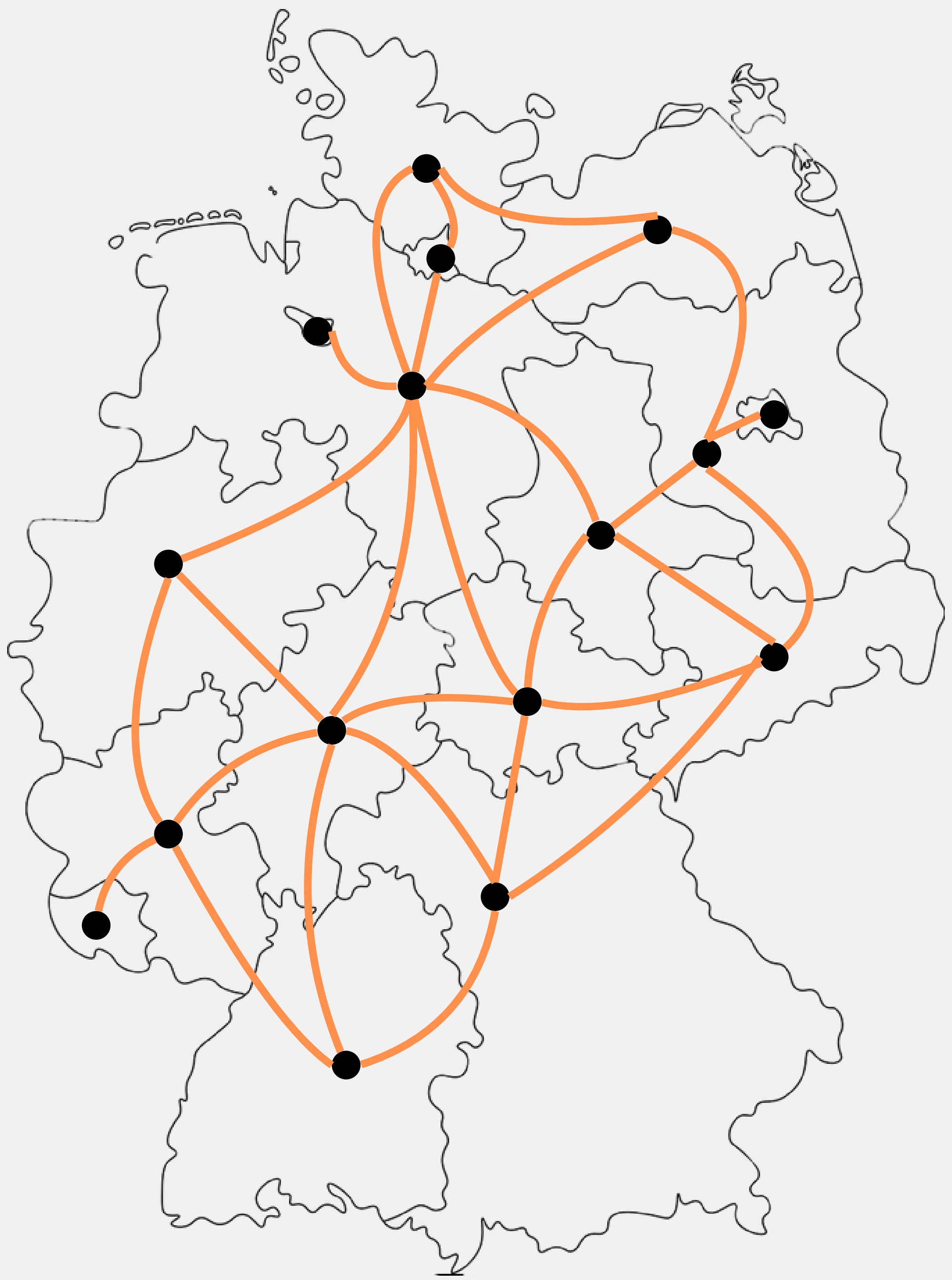
The Four Colour Conjecture (1852)



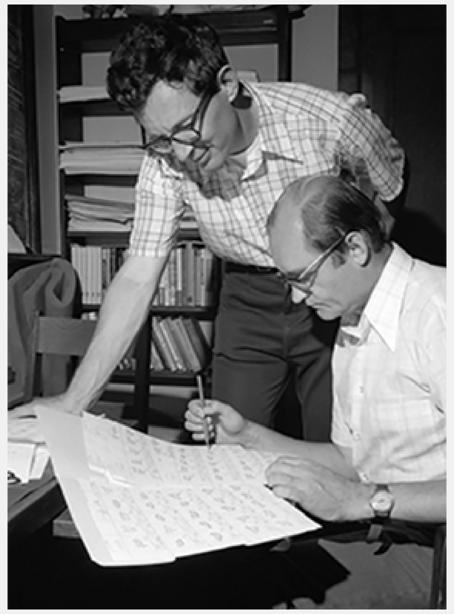


COLOURING PROBLEMS

The Four Colour Theorem (1976)



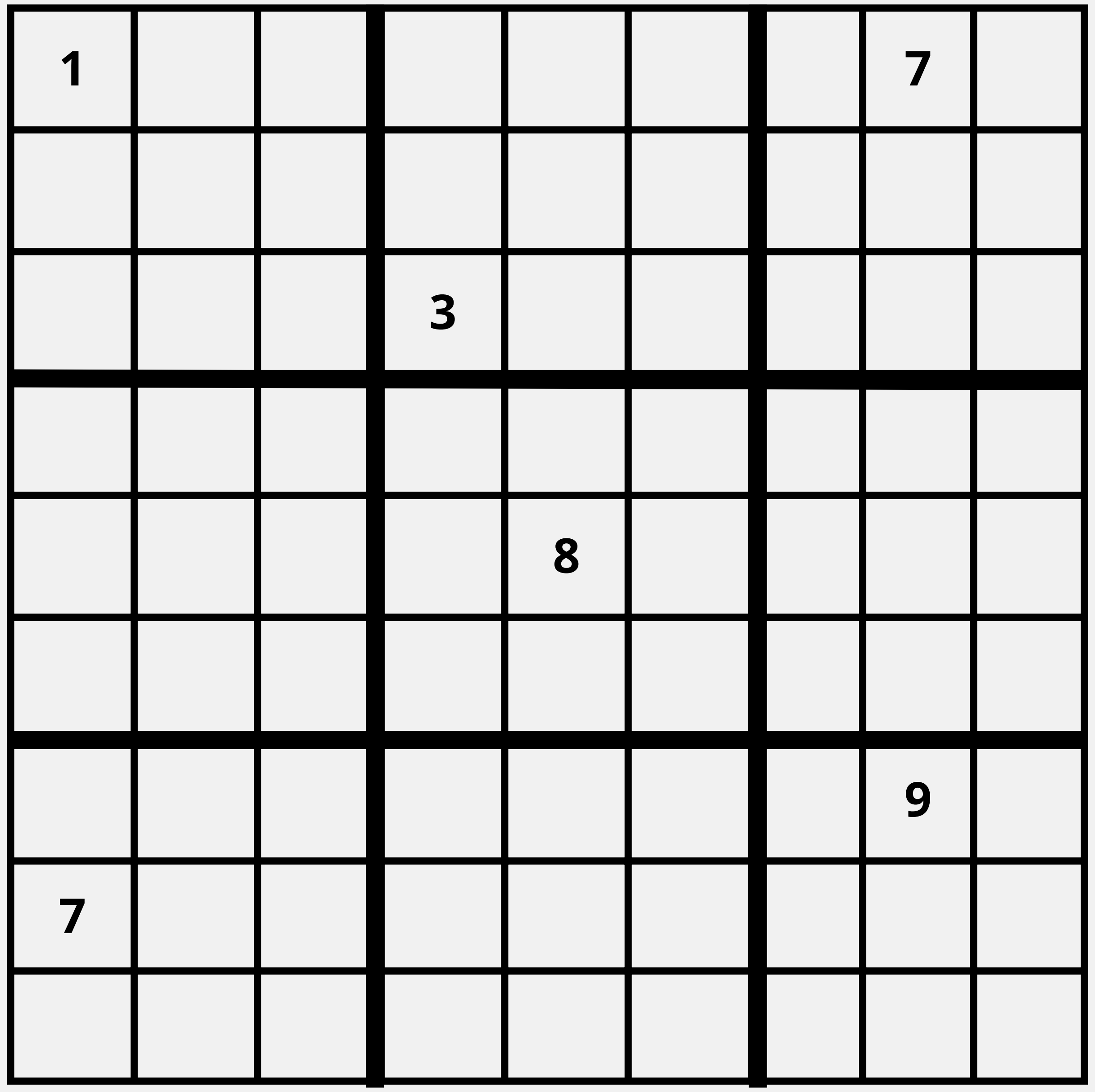
Every map can be coloured with four colours in such a was that neighbouring regions have different colours

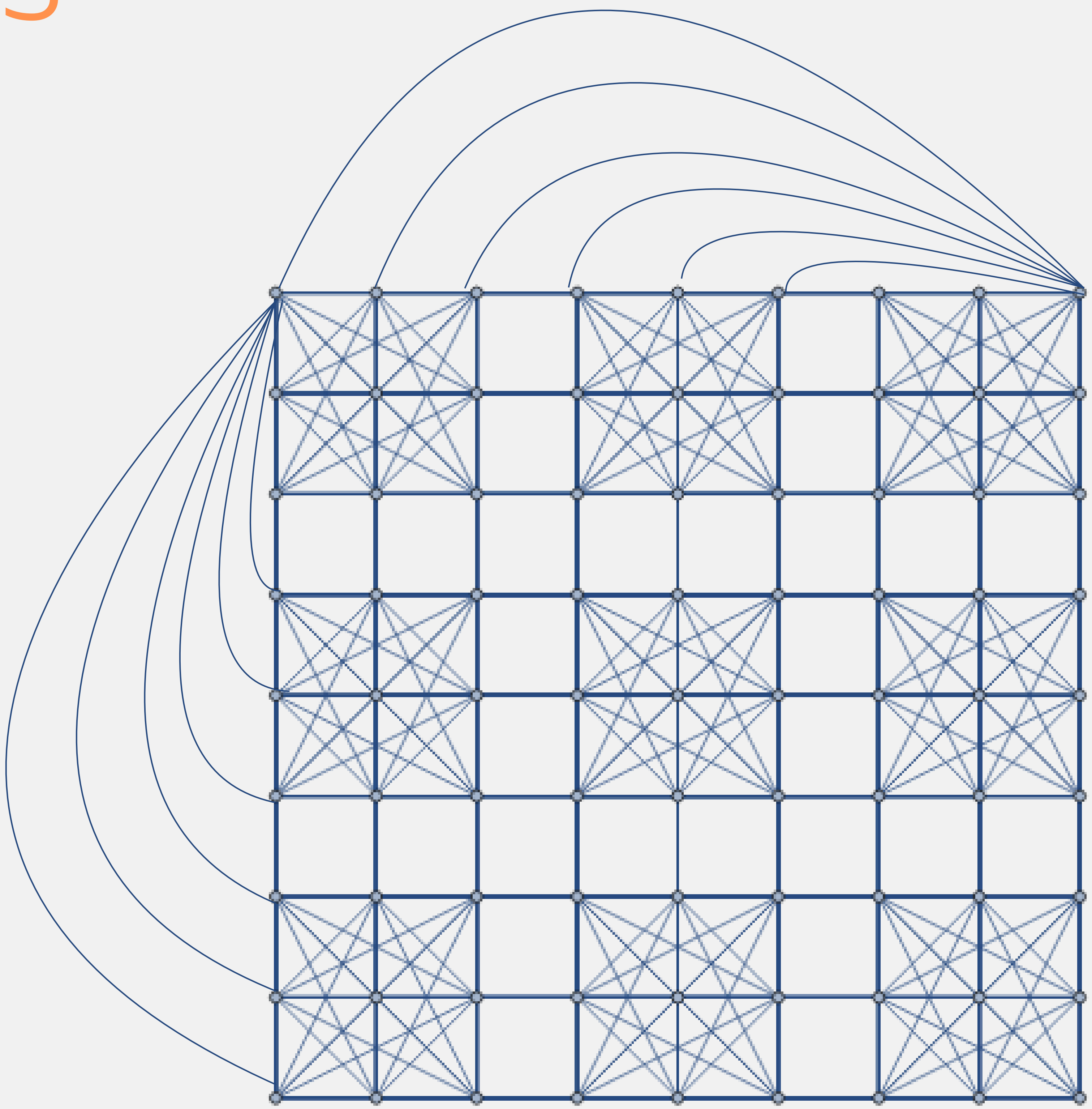


Appel and Hakken

COLOURING PROBLEMS

Sudoku





Source: Wolfram Community

COLOURING PROBLEMS

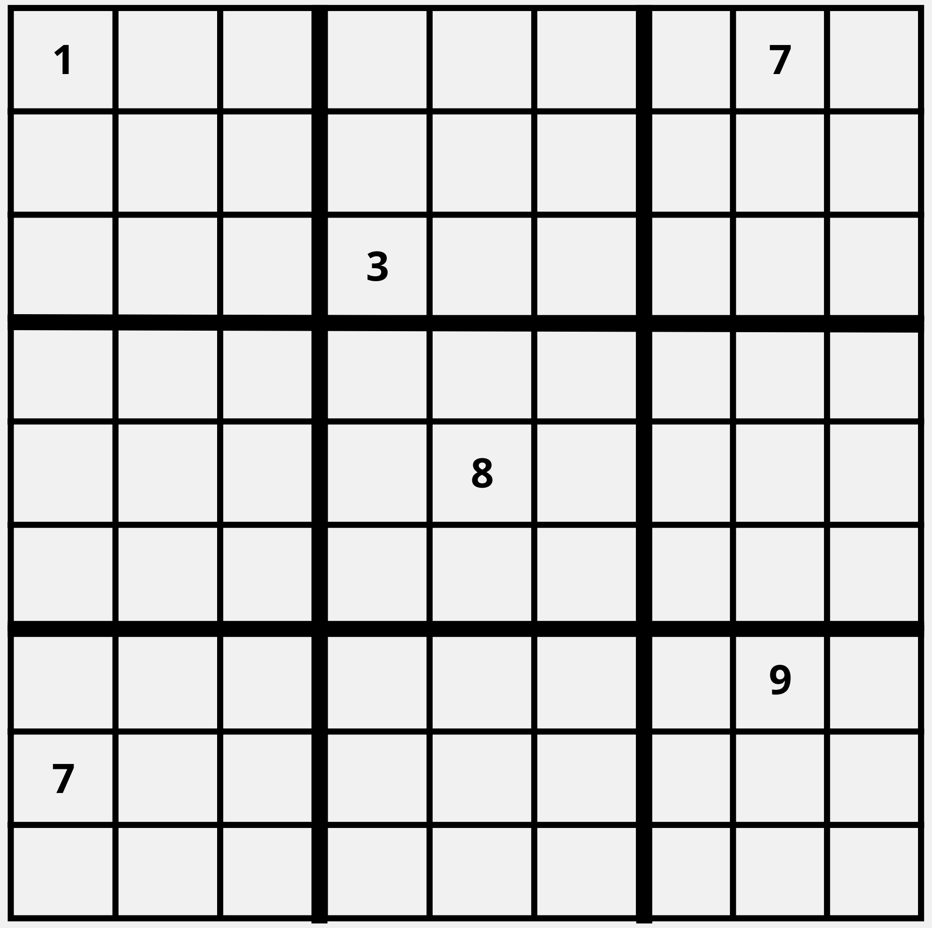
Computational complexity

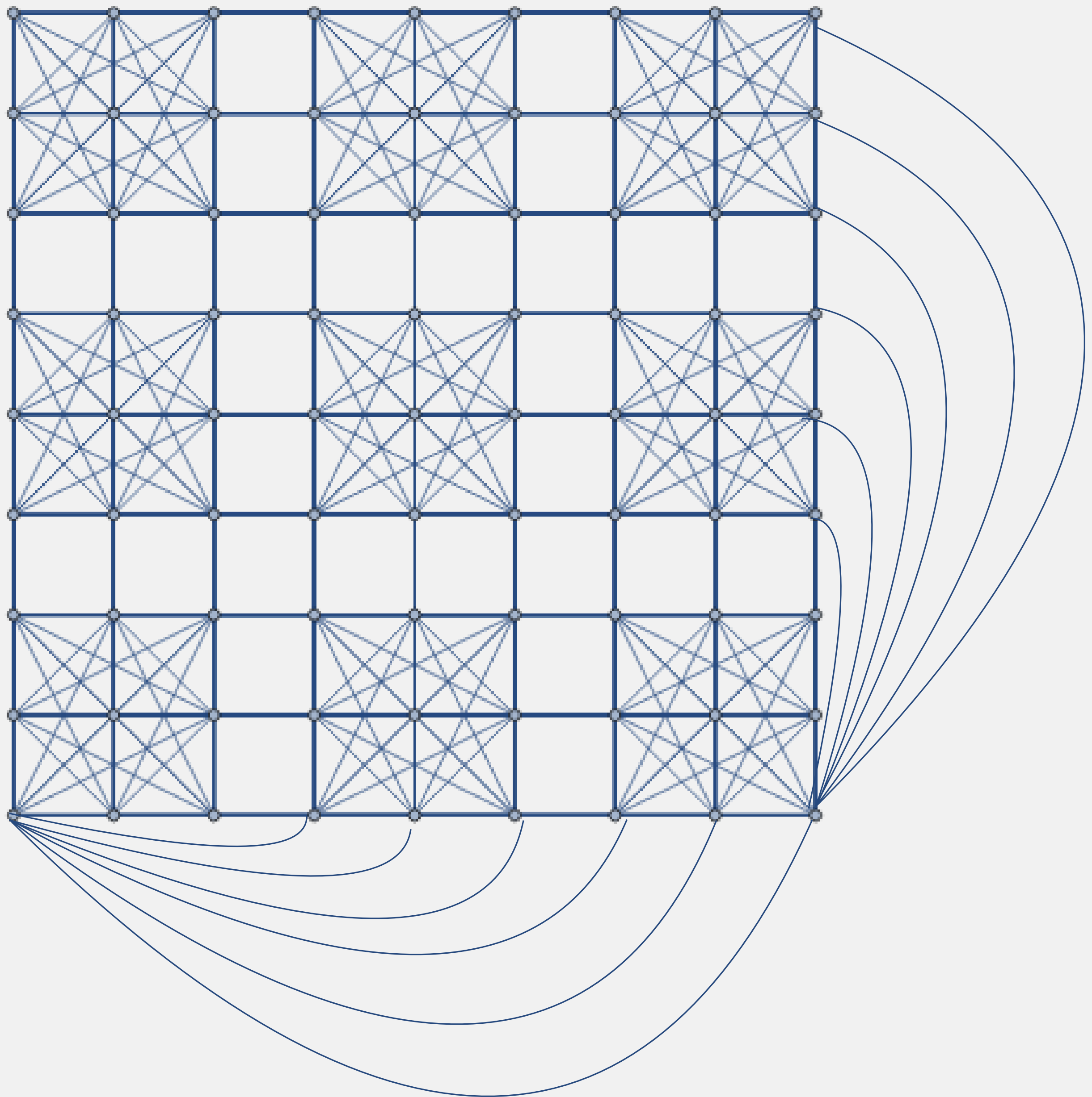
Given an ***input*** graph G determine whether there is a ***proper k-colouring*** of G

COLOURING PROBLEMS

Computational complexity

Given an ***input*** graph G determine whether there is a ***proper 9-colouring*** of G

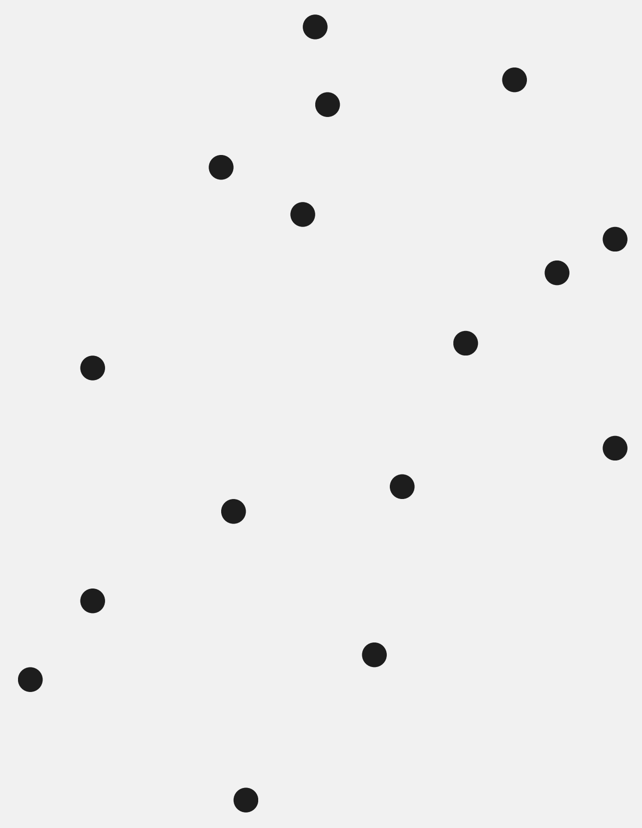


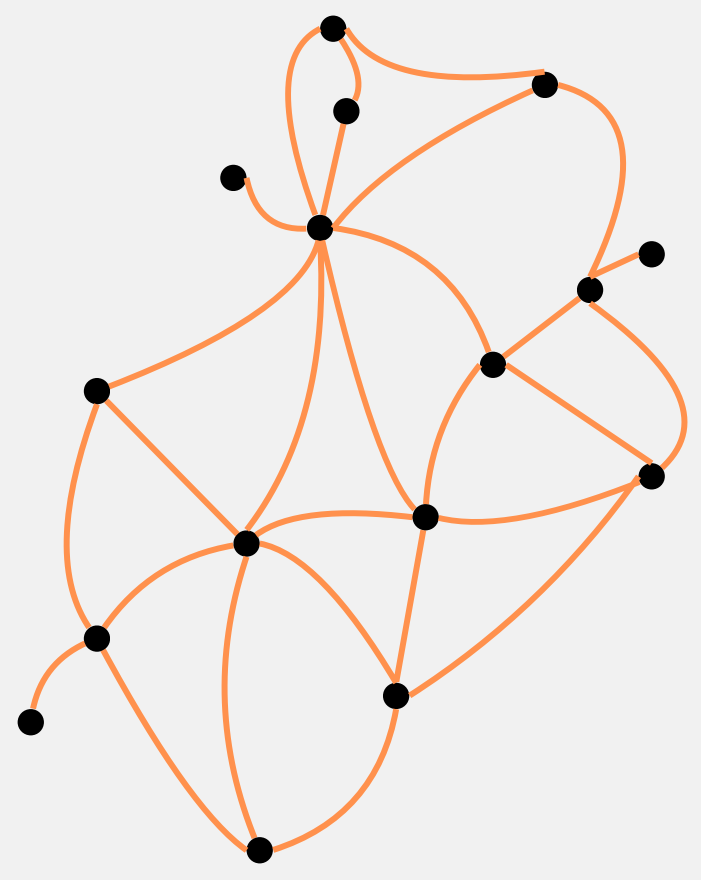


COLOURING PROBLEMS

Computational complexity

Given an ***input*** graph G determine whether there is a ***proper 1-colouring*** of G

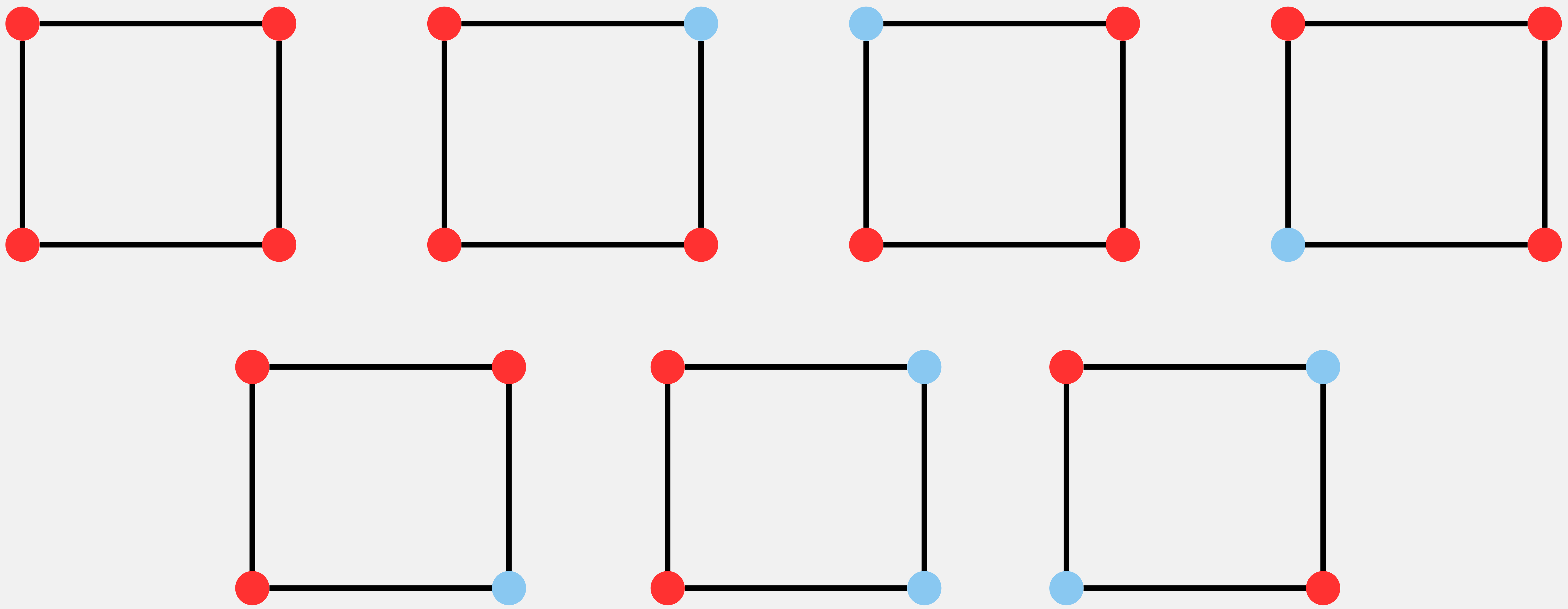




COLOURING PROBLEMS

Computational complexity

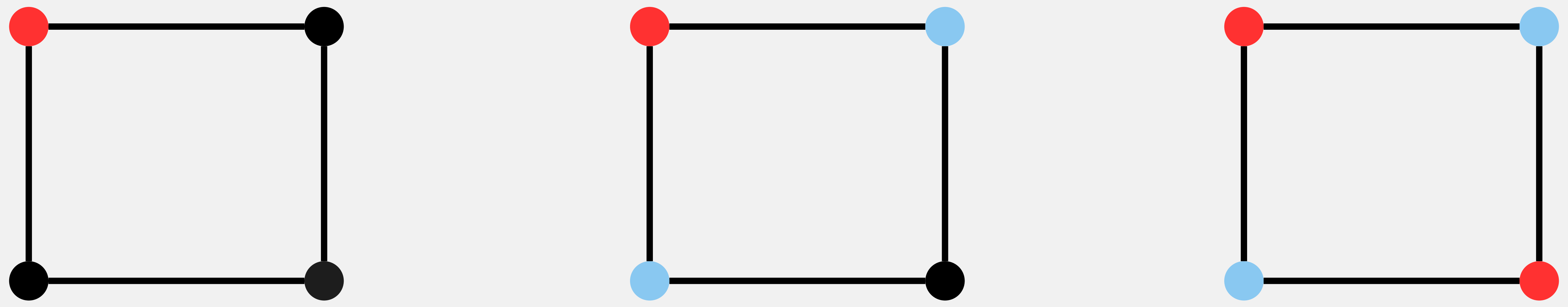
Given an ***input*** graph G determine whether there is a ***proper 2-colouring*** of G



COLOURING PROBLEMS

Computational complexity

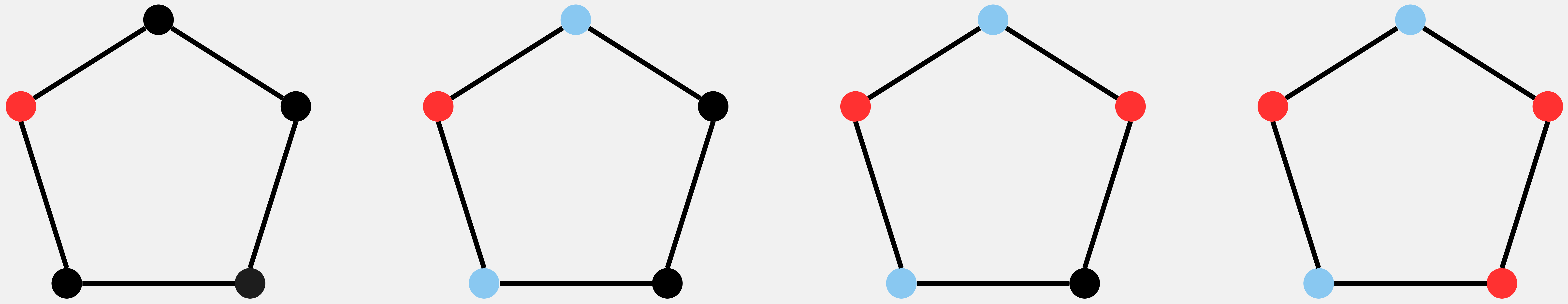
Given an ***input*** graph G determine whether there is a ***proper 2-colouring*** of G



COLOURING PROBLEMS

Computational complexity

Given an ***input*** graph G determine whether there is a ***proper 2-colouring*** of G



COLOURING PROBLEMS

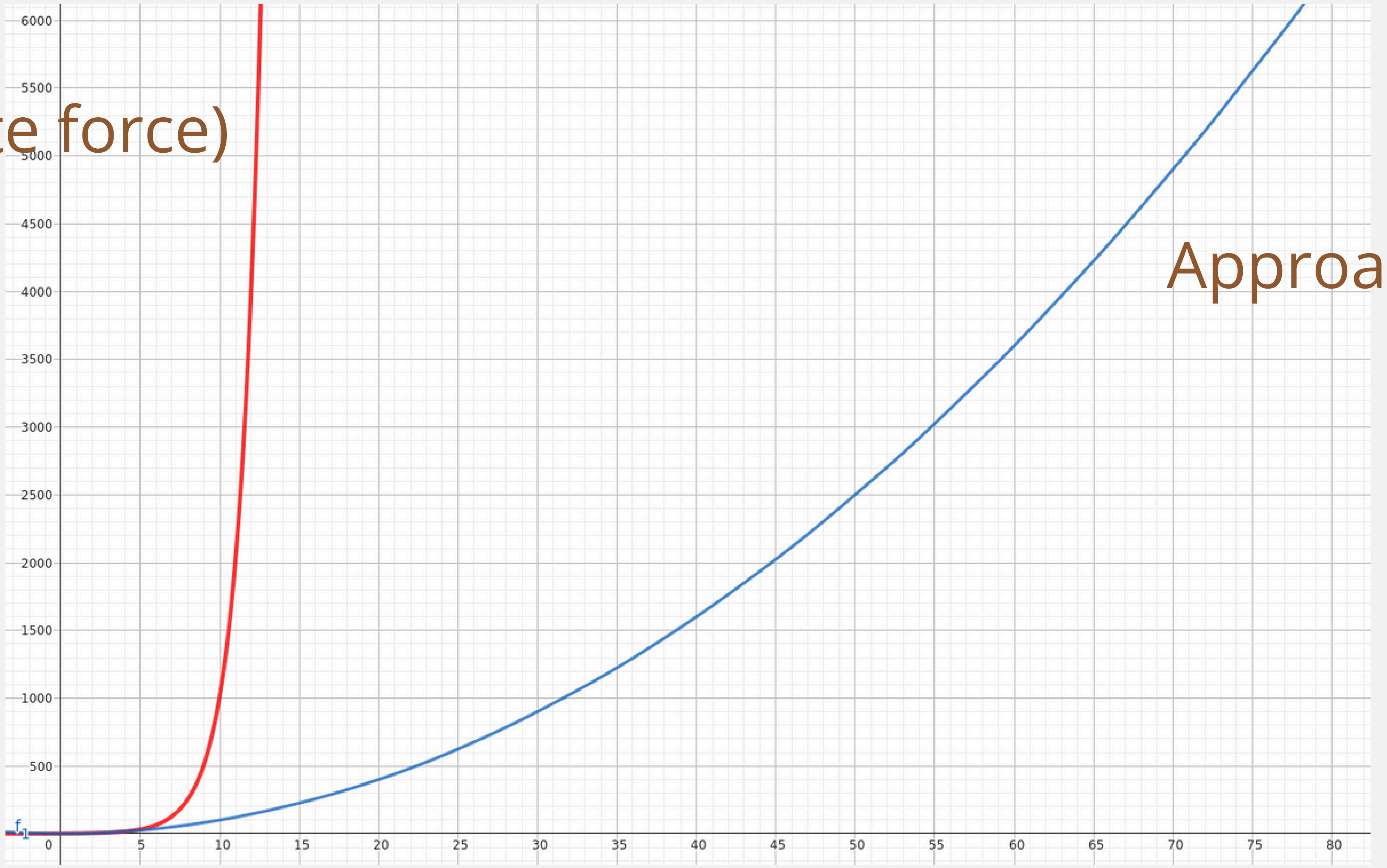
Computational complexity

Given an ***input*** graph G determine whether there is a ***proper 2-colouring*** of G

Approach 1 (Brute force)

Running time: 2^n

2^{70} seconds > age of the universe



Approach 2 (Consistency)

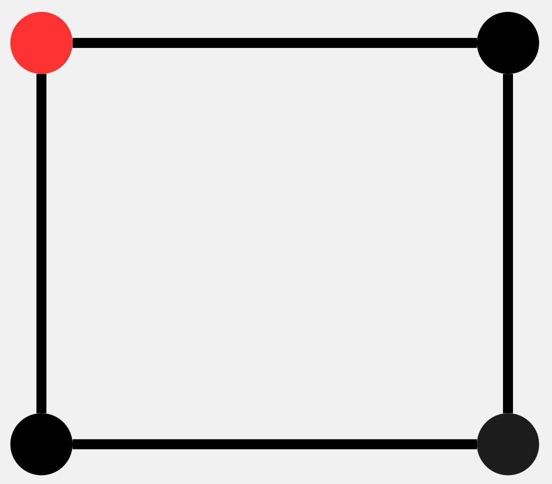
Running time: n^2

70^2 seconds < 1h30'

COLOURING PROBLEMS

Computational complexity

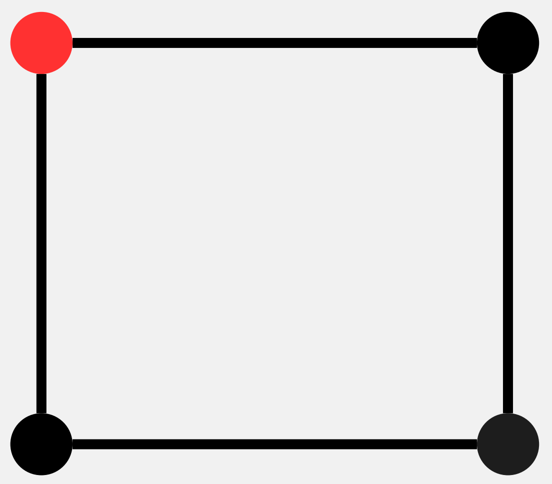
Given an ***input*** graph G determine whether there is a ***proper 3-colouring*** of G

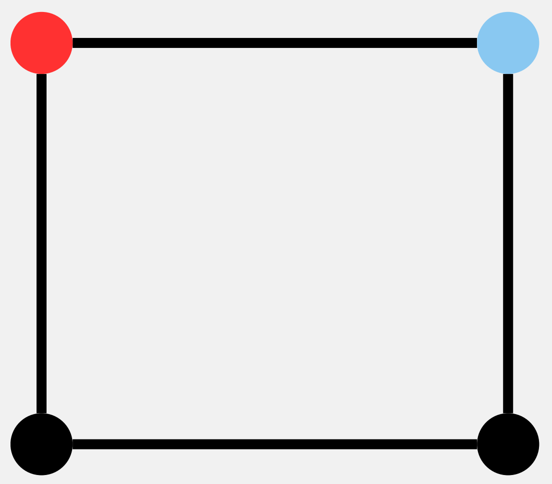


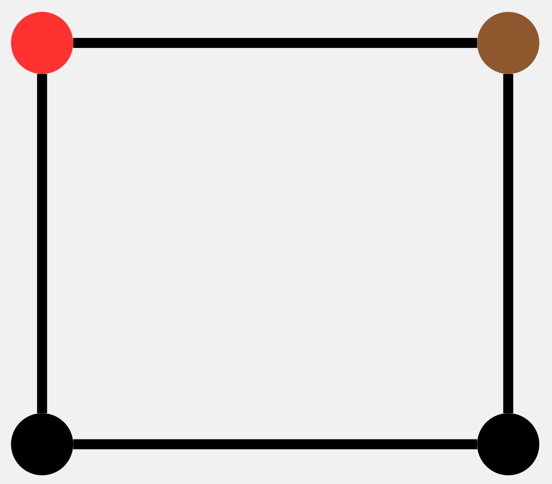
COLOURING PROBLEMS

Computational complexity

Given an ***input*** graph G determine whether there is a ***proper 3-colouring*** of G



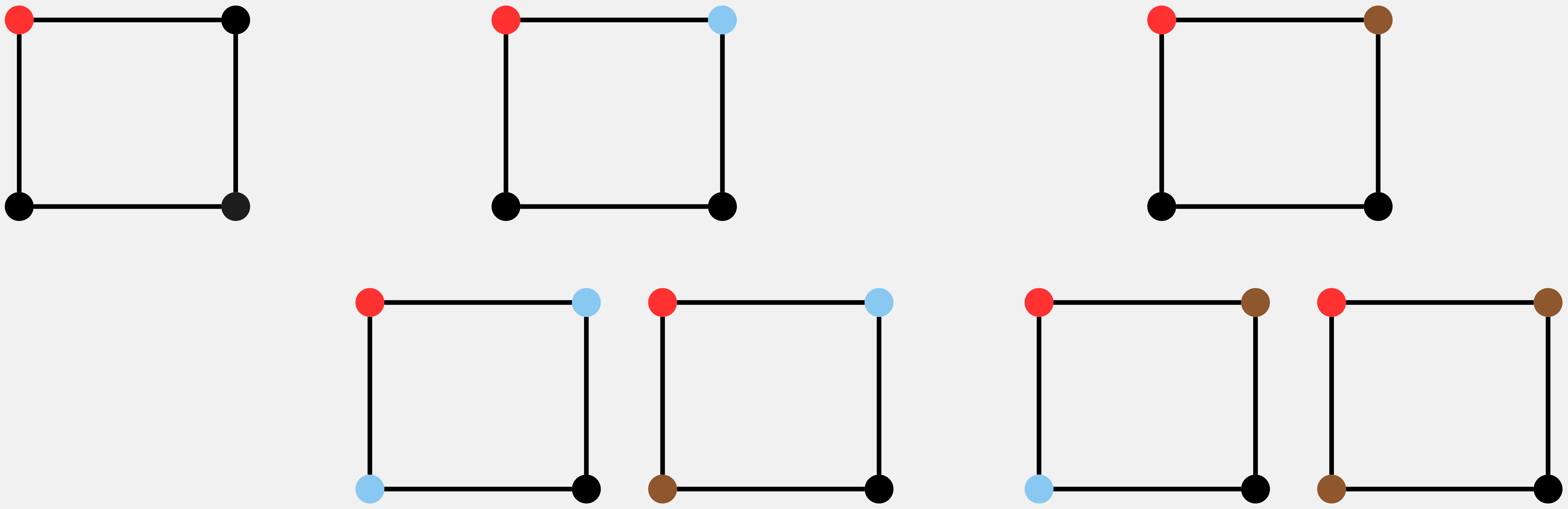




COLOURING PROBLEMS

Computational complexity

Given an ***input*** graph G determine whether there is a ***proper 3-colouring*** of G



COLOURING PROBLEMS

Computational complexity

Given an ***input*** graph G determine whether there is a ***proper 3-colouring*** of G

Approach 1 and 2 give exponential running times :(

Is there an efficient algorithm that solves 3-colouring?

COLOURING PROBLEMS

Computational complexity

Given an ***input*** graph G determine whether there is a ***proper 3-colouring*** of G

Approach 1 and 2 give exponential running times :(

Is there an efficient algorithm that solves 3-colouring?
(this is a **million** dollar question)

COLOURING PROBLEMS

The P versus NP problem: a Millennium Prize Problem

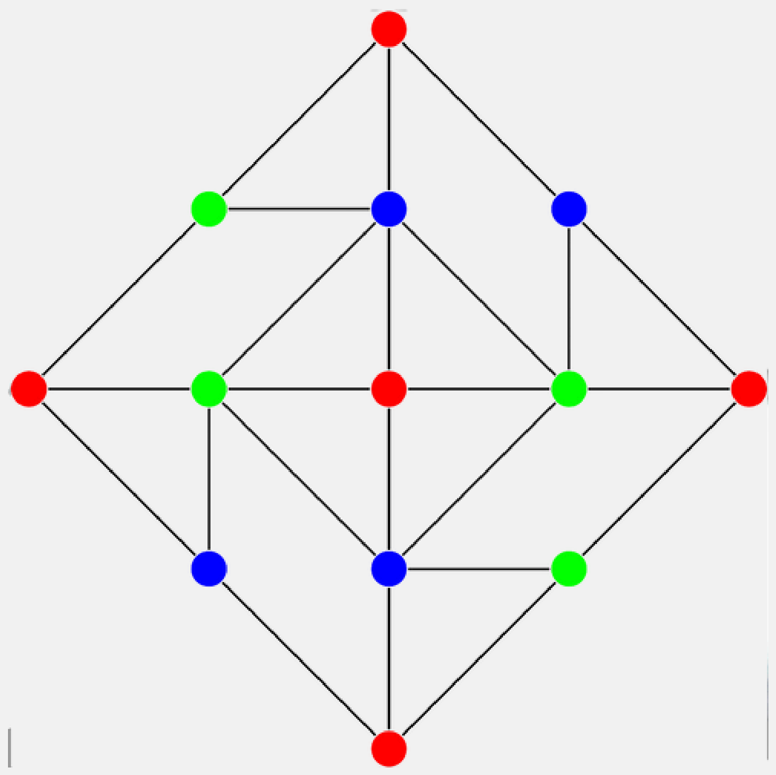
If the solution to a problem is *easy* to ***verify***, is the problem also *easy* to ***solve***?

COLOURING PROBLEMS

The P versus NP problem: a Millennium Prize Problem

If the solution to a problem is *easy* to ***verify***, is the problem also *easy* to ***solve***?

3-colouring

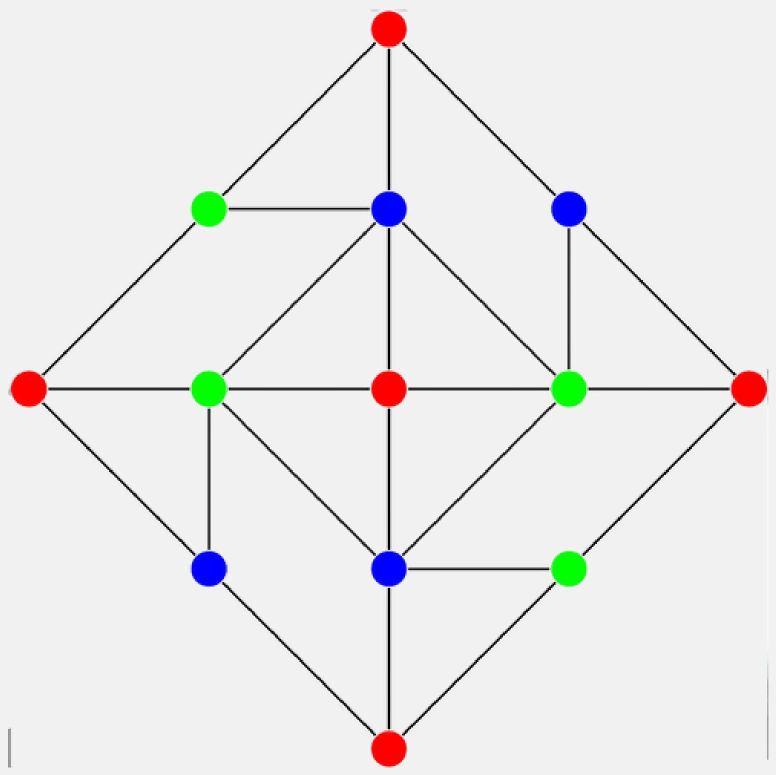


COLOURING PROBLEMS

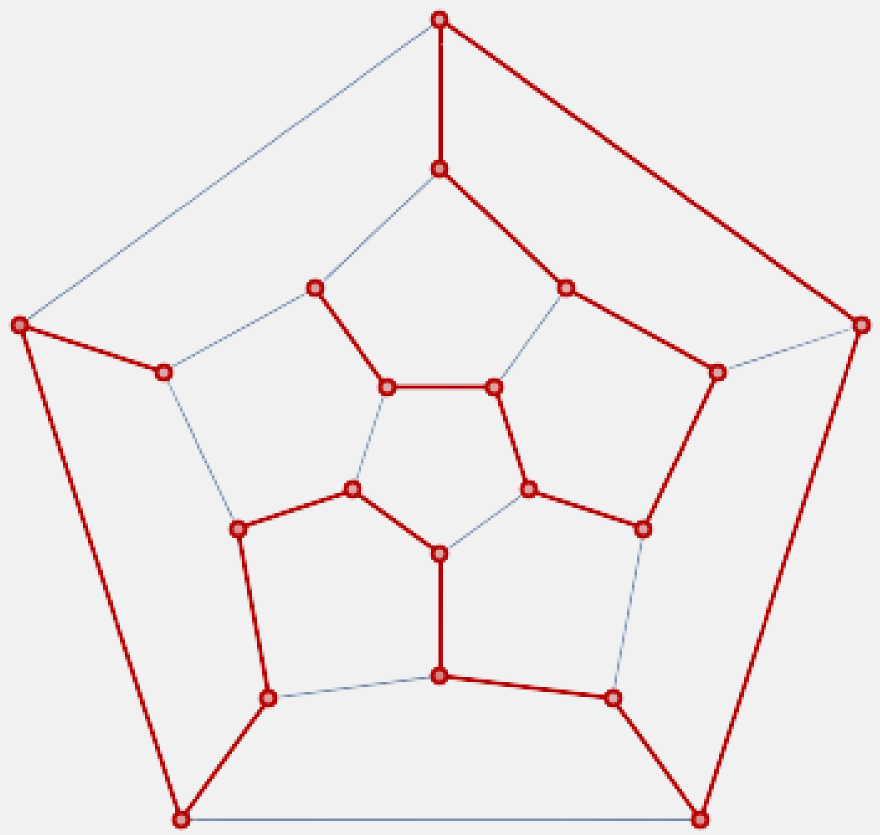
The P versus NP problem: a Millennium Prize Problem

If the solution to a problem is *easy* to ***verify***, is the problem also *easy* to ***solve***?

Graph colouring



Hamiltonian path

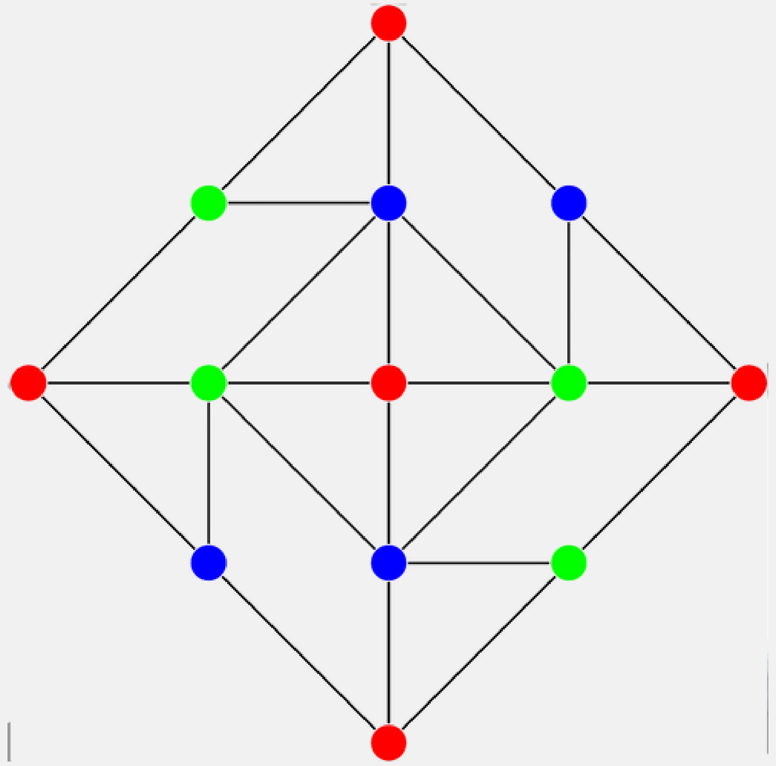


COLOURING PROBLEMS

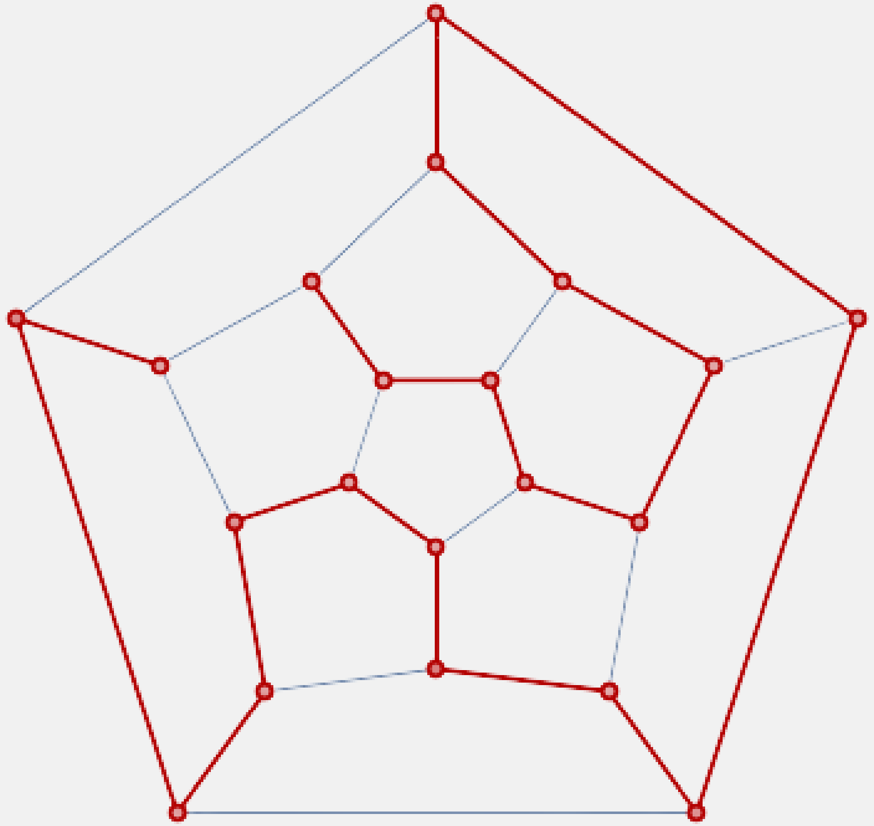
The P versus NP problem: a Millennium Prize Problem

If the solution to a problem is *easy* to ***verify***, is the problem also *easy* to ***solve***?

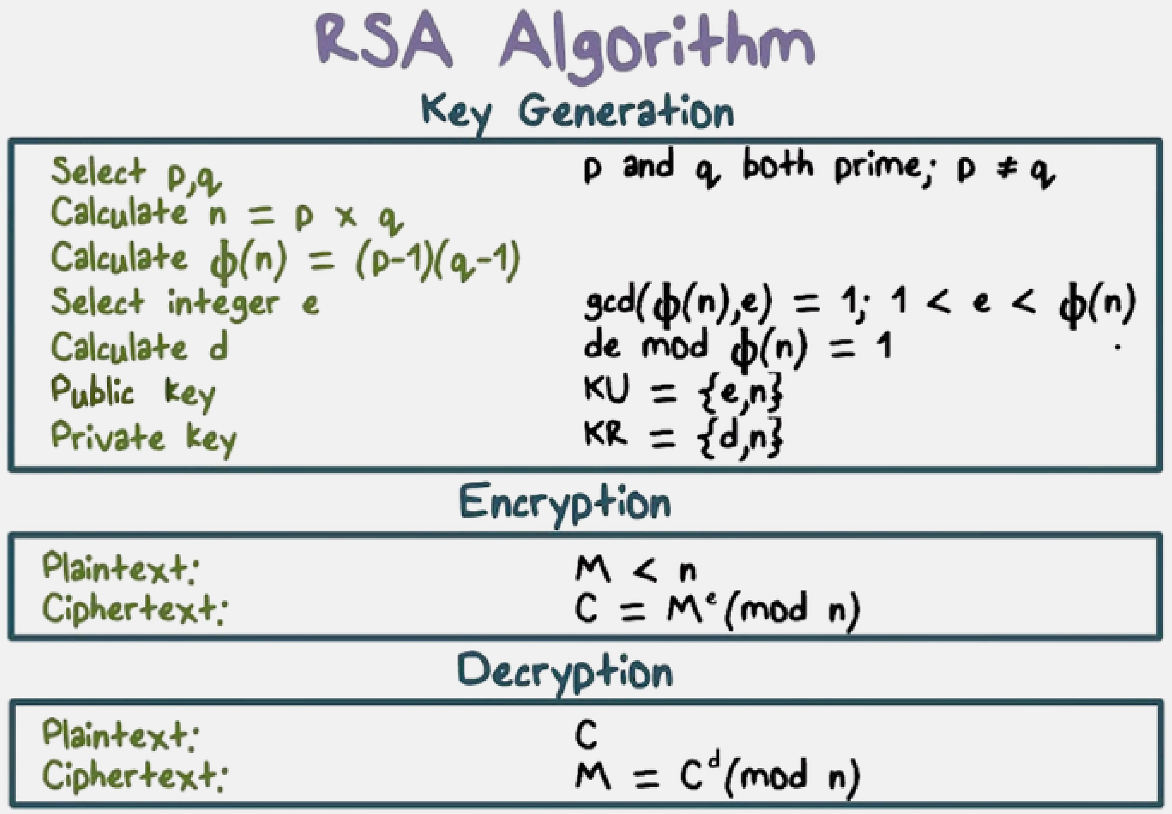
Graph colouring



Hamiltonian path



Integer Factorization



THANK YOU FOR YOUR ATTENTION!