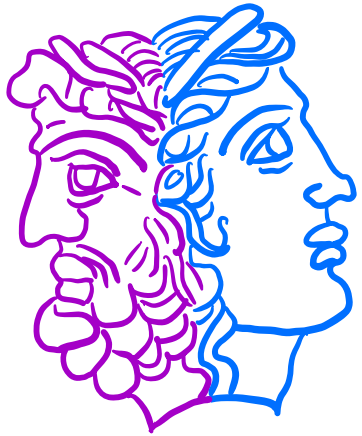
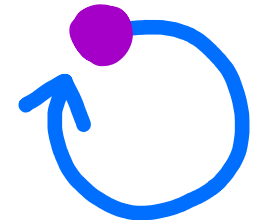


IANUS-FACES OF TEMPORAL CSPs A DICHOTOMY OF EXPRESSIVITY



Johanna Brunar
TU Wien



joint work with M. Pinsker, M. Schöbi



ÖAW

ÖSTERREICHISCHE
AKADEMIE DER
WISSENSCHAFTEN

MFO, Dec. 2025

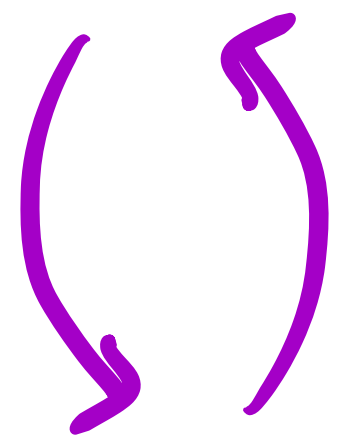
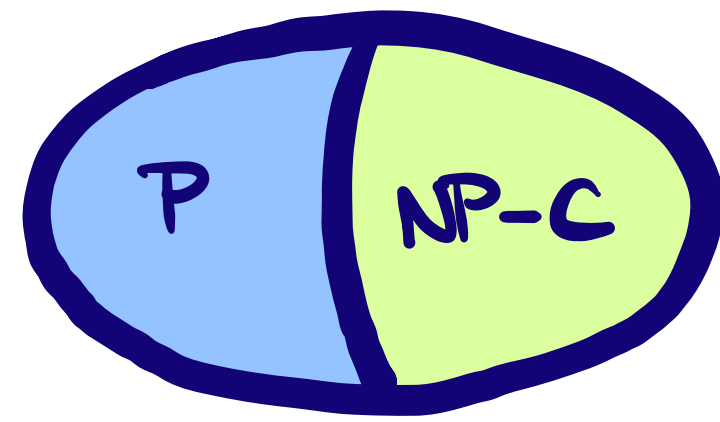
ALGEBRAIC APPROACH

JEAVONS '38
BULATOV+
JEAVONS+
KROKHIN '05
BARTO+OPRŠAL+
PINSKER '18

$\mathcal{A}$ FINITE

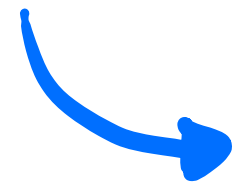
$\text{CSP}(\mathcal{A})$

...

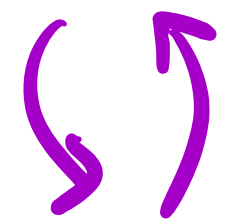
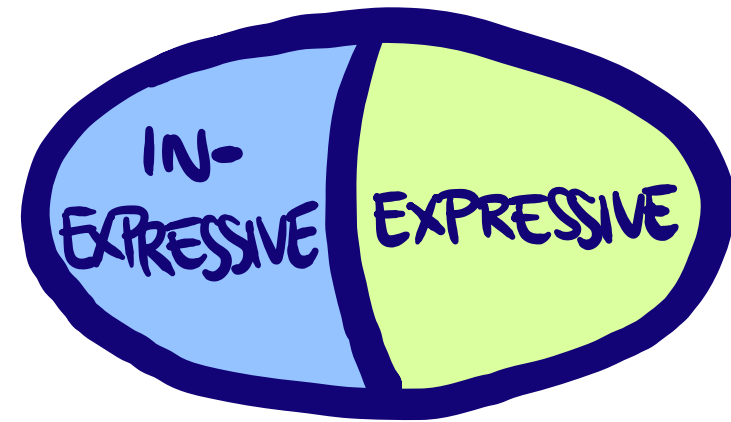


$\mathcal{A}$

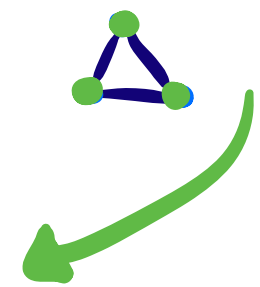
SYMMETRIES
LOOPS



...



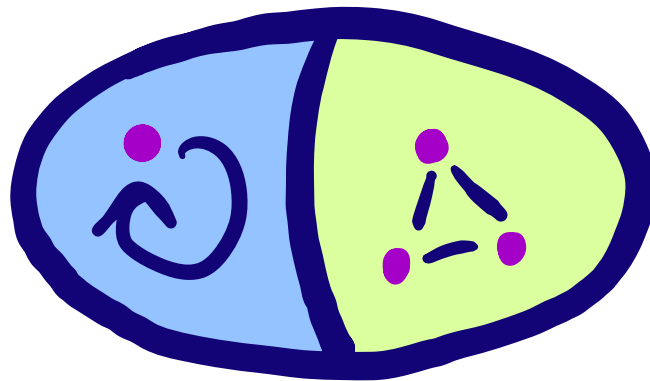
EXPRESS
STH HARD



ALGEBRAIC APPROACH

THM (HELL + NEŠETŘIL '90)

$\mathbb{Q}$ NON-BIPARTITE FINITE GRAPH.
EITHER $\mathbb{Q}$ IS OMNI-EXPRESSIVE
OR $\mathbb{Q}$ HAS A LOOP.

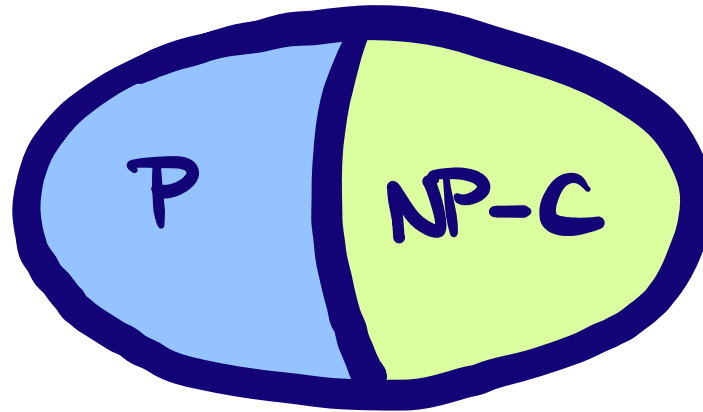


PP-CONSTRUCT
ALL FINITE
STRUCTURES

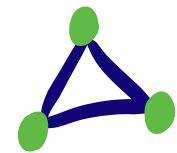
ALGEBRAIC DICHOTOMY

A FINITE

EQUATIONAL
STRUCTURE
OF $\text{POL}(A)$



A OMNI-
EXPRESSIVE



$$s(x, y, x, z, y, z) \approx s(y, x, z, x, z, y)$$

$$s(d, r, e, a) \approx s(r, a, r, e)$$

$$c(x_1 \dots x_p) \approx c(x_2 \dots x_p, x_1)$$

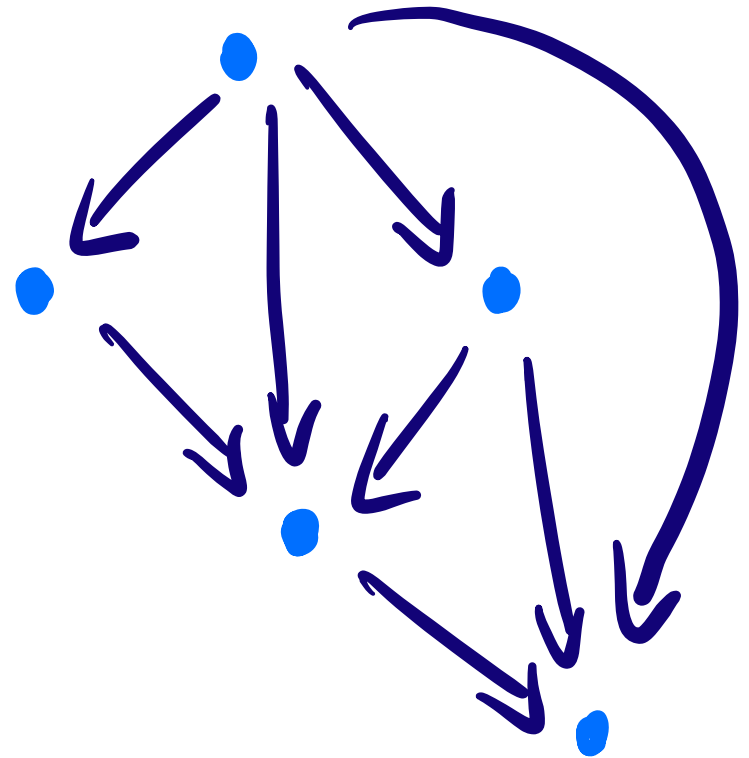
FROM FINITE
TO INFINITE

FROM FINITE TO INFINITE

$\text{CSP}(\mathbb{Q}; <)$

INPUT: $\times$ FINITE
DIGRAPH

QUESTION: IS $\times$
ACYCLIC?



ALGEBRAIC APPROACH

(BODIRSKY+PINSKER '15)

$\mathcal{A}$ W-CAT. $\leadsto$ 'LOCAL' EQUATIONAL
STRUCTURE OF $\text{POL}(\mathcal{A})$.

ALGEBRAIC APPROACH

(BODIRSKY + PINSKER '15)

$\mathcal{A}$ ω -CAT. $\leadsto$ 'LOCAL' EQUATIONAL
STRUCTURE OF $\text{POL}(\mathcal{A})$.

Thm (BARTO + PINSKER '21)

$\mathcal{A}$ ω -CAT, NOT OMNI-EXPRESSIVE. THEN

$$\text{POL}(\mathcal{A}) \models u \circ S(x, y, x, z, y, z) \approx v \circ S(y, x, z, x, z, y)$$

ALGEBRAIC APPROACH

(BODIRSKY + PINSKER '15)

A ω -CAT. $\leadsto$ 'LOCAL' EQUATIONAL
STRUCTURE OF $\text{POL}(A)$.

Thm (BARTO + PINSKER '21)

A ω -CAT, NOT OMNI-EXPRESSIVE. THEN

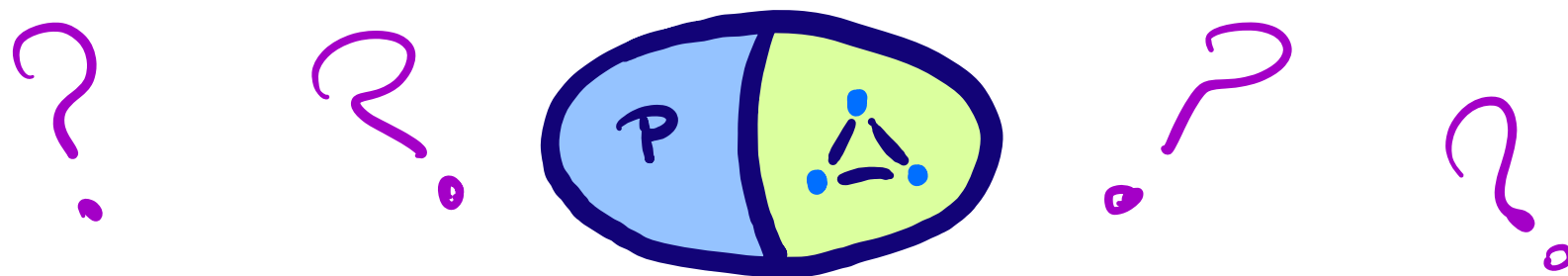
$$\text{POL}(A) \models u \circ S(x, y, x, z, y, z) \approx v \circ S(y, x, z, x, z, y)$$

WHAT ELSE?

INFINITE DICHOTOMY CONJECTURE

CONJ: (BODIRSKY + PINSKER '12)

A F.O. REDUCT OF A FINITELY
BOUNDED HOMOGENEOUS STRUCTURE.
THEN CSP(A) IS SOLVABLE IN
P-TIME OR A IS OMNI-EXPRESSIVE.



INFINITE DICHOTOMY CONJECTURE

VERIFIED FOR REDUCTS OF

- $(\mathbb{Q}, <)$

BODIRSKY + KÁRA '07

- HOMOGENEOUS GRAPHS

BODIRSKY + PINSKER '11

BODIRSKY + MARTIN +
PINSKER + PONGRÁCZ '15

- GENERIC POSET

KOMPATSCHER +
VAN PHAM '16

- GENERIC TOURNAMENT

MOTTET + PINSKER '21

- GENERIC HYPERGRAPH

MOTTET + NAGY +
PINSKER '23

REDUCTION TO FINITE

BODIRSKY + NOTTET '16

CANONICAL POLYMORPHISMS

↖ ACT ON ORBITS

→ TRACTABILITY CONDITION

→ IDENTITIES LIFT

TEMPORAL CONSTRAINT LANGUAGES

= F.O. REDUCTS OF $(\mathbb{Q}; <)$

- NO NON-TRIVIAL CANONICAL POLS

TEMPORAL CONSTRAINT LANGUAGES

= F.O. REDUCTS OF $(\mathbb{Q}; <)$

- NO NON-TRIVIAL CANONICAL POLS

- ALGORITHMICALLY DIFFERENT

- DESCRIPTIVE COMPLEXITY

- ALGEBRAIC PROPERTIES

IDENTITIES!

BODIRSKY +
KÁRA '08,
MOTTET '25
ZHUK '25

BODIRSKY +
RYDVAL '22

B. + PINSKER +
SCHÖBI '25

LOOP
LEMMATA

LOOP LEMMATA

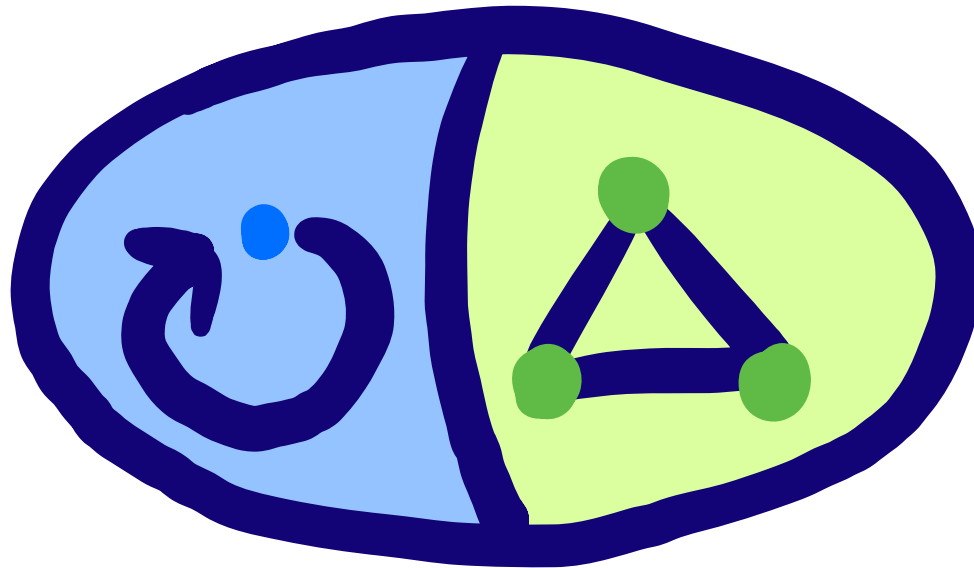
Thm: (HELL+NEŠETŘIL '90)

⊆ FINITE **UNDIRECTED**
NON-BIPARTITE.

Thm: (BARTO+KŮŽIK+NIVEN '03)

⊆ FINITE **DIRECTED** SMOOTH
ALGEBRAIC LENGTH 1.

LOOP



OMNI-
EXPRESSIVE

LOOP LEMMATA

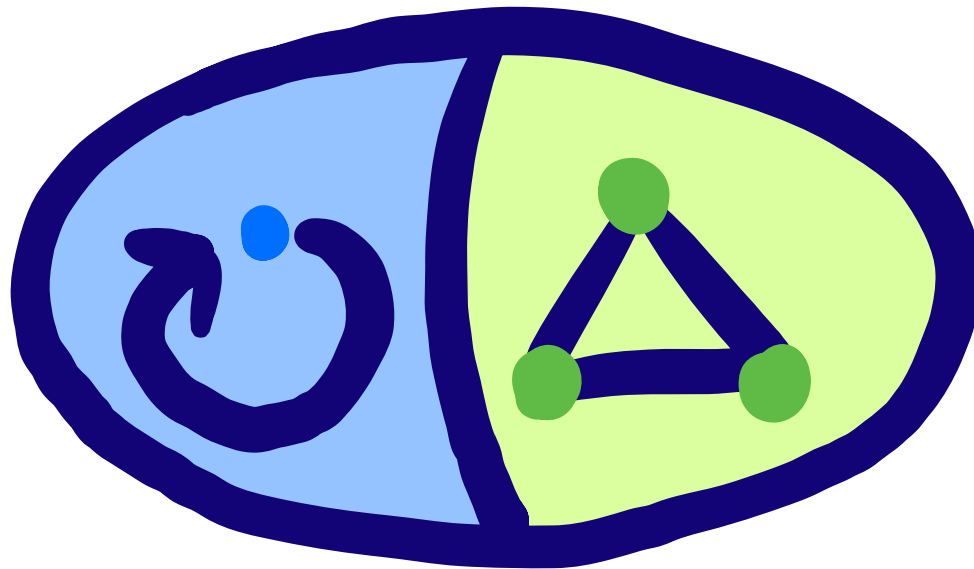
Thm: (HELL+NEŠETŘIL '80)

⊆ FINITE **UNDIRECTED**
NON-BIPARTITE.

Thm: (BARTO+KŮŽIK+NIVEN '03)

⊆ FINITE **DIRECTED** SMOOTH
ALGEBRAIC LENGTH 1.

P



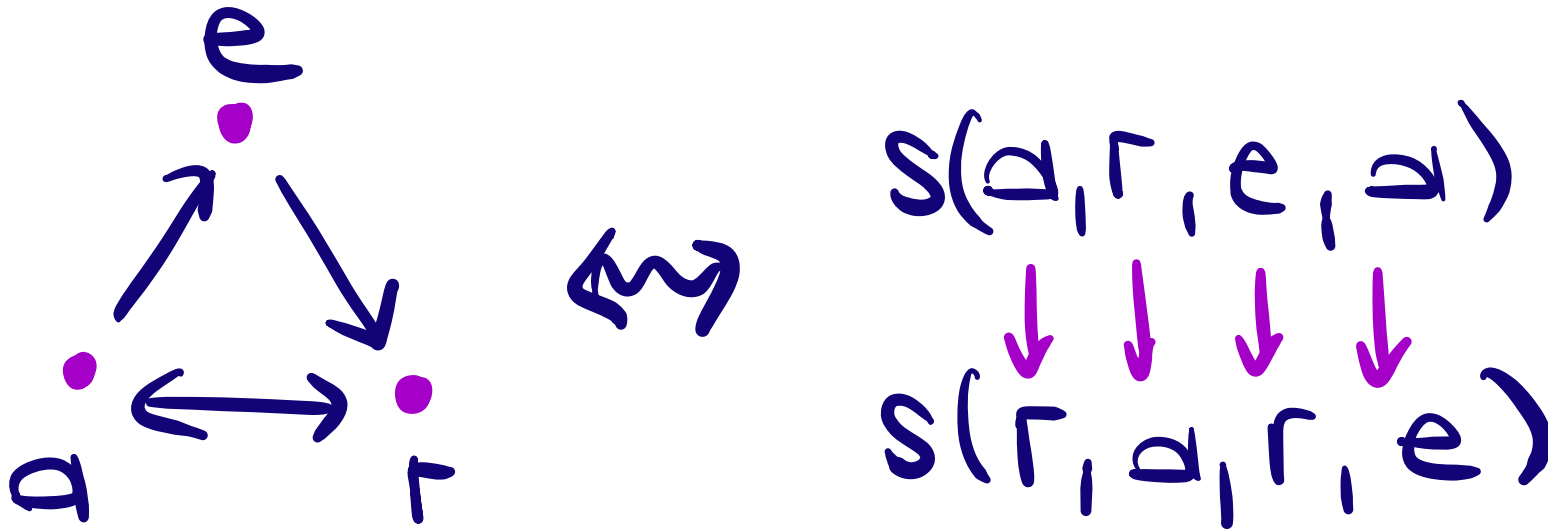
NP-C

LOOPS $\Leftrightarrow$ IDENTITIES

SIGGERS '10

BARTO+KOZIK+
NIVEN '03

KEARNES+
MARKOVIC+
MCKENZIE '15



$$\mathbb{F} = \left\langle \begin{pmatrix} a \\ r \end{pmatrix} \begin{pmatrix} r \\ a \end{pmatrix} \begin{pmatrix} e \\ r \end{pmatrix} \begin{pmatrix} a \\ e \end{pmatrix} \right\rangle_{\text{POL}(\mathbb{A})}$$

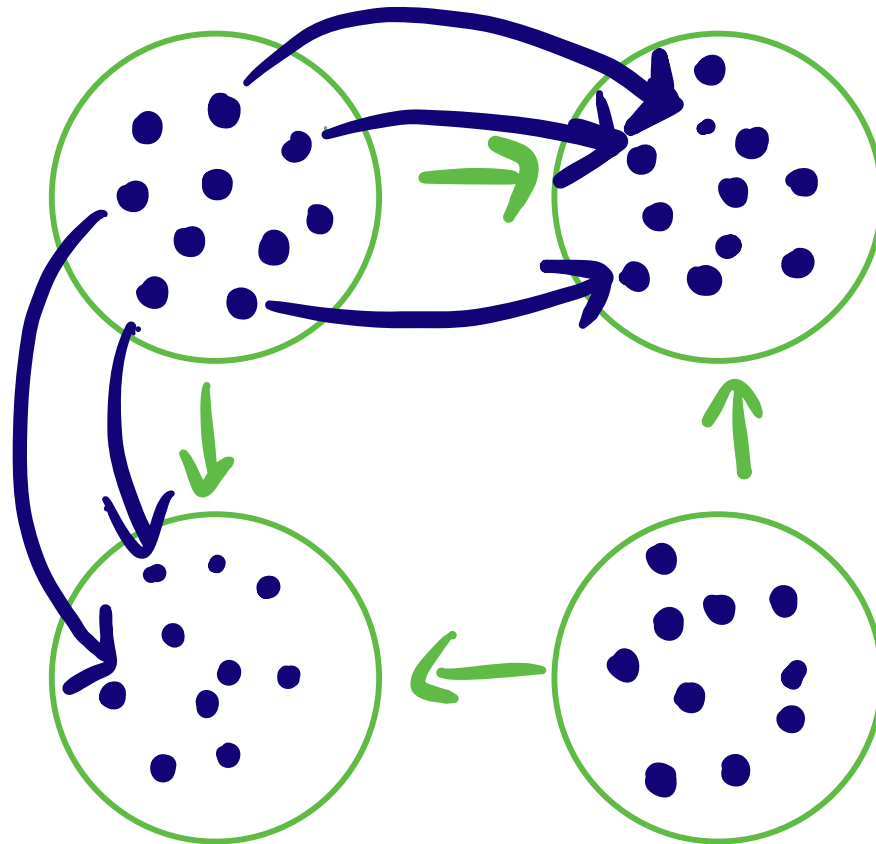
IS OMNI-EXPRESSIVE OR HAS LOOP.

CLOSE TO FINITE: ω -CATEGORICITY

$\mathbb{E}$ ω -CATEGORICAL DIGRAPH

FINITE FACTOR:

$\mathbb{E}/\text{Aut}(\mathbb{E})$



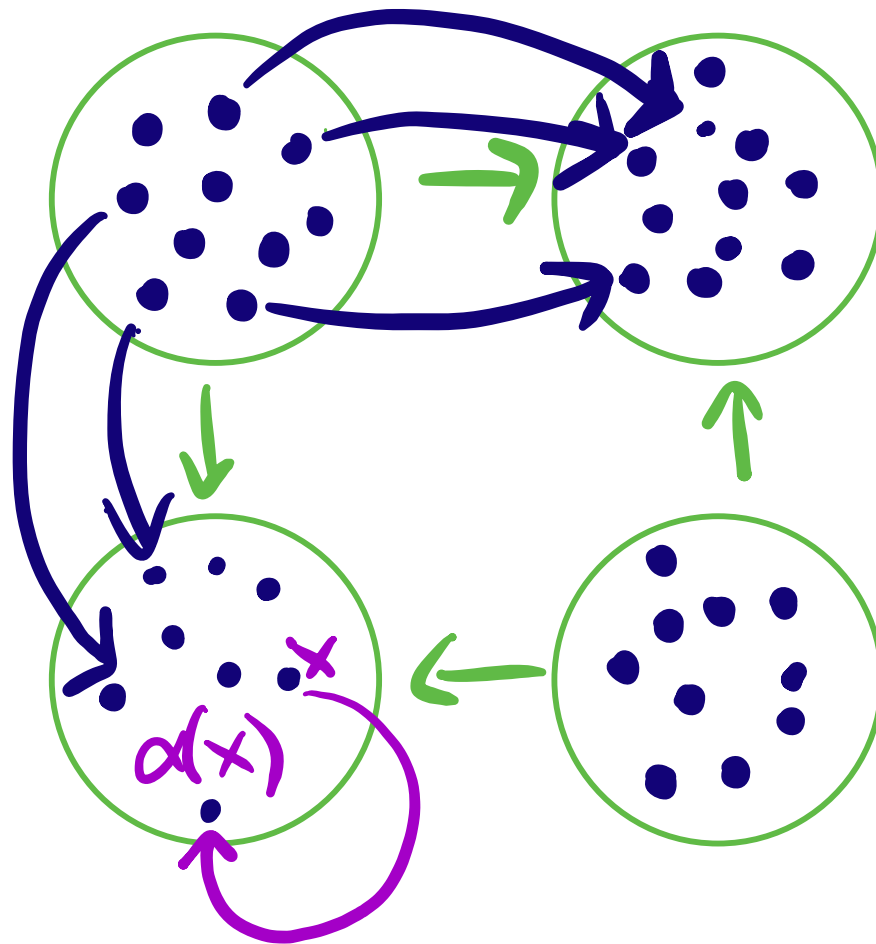
CLOSE TO FINITE: ω -CATEGORICITY

$\mathbb{E}$ ω -CATEGORICAL DIGRAPH

FINITE FACTOR:

$\mathbb{E}/\text{Aut}(\mathbb{E})$

PSEUDO-LOOP



INFINITE HELL-NESETRIL

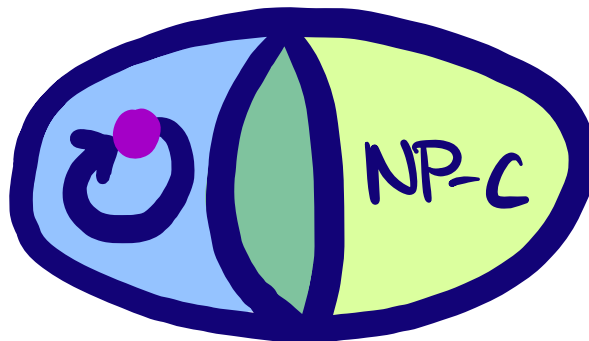
Thm (BARTO + BODOR + KOZIK + MOTTET + PINSKER '23)

$\mathbb{G}$ SMOOTH, ω -CATEGORICAL

$\mathbb{G}/\text{Aut}(\mathbb{G})$ SYMMETRIC, NON-BIPARTITE.

EITHER $\mathbb{G}$ IS OMNI-EXPRESSIVE

OR $\mathbb{G}$ HAS PSEUDO-LOOP.

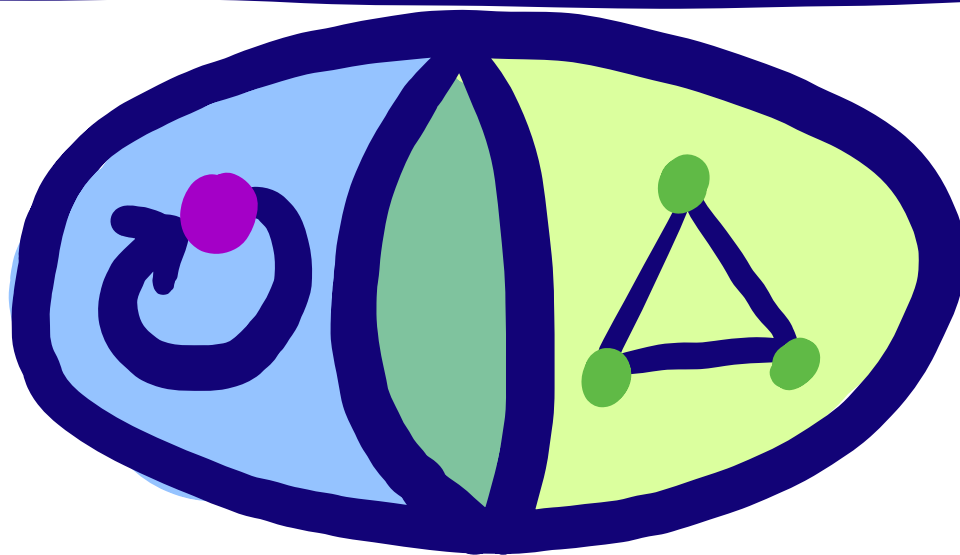


INFINITE BARTO-KOZIK-NIVEN?

Thm (B. + KOZIK + NAGY + PINSKER '25)

⊢ SMOOTH DIGRAPH OF ALGEBRAIC LENGTH 1
CONSERVATIVE, ω -CATEGORICAL

PSEUDO-
LOOP



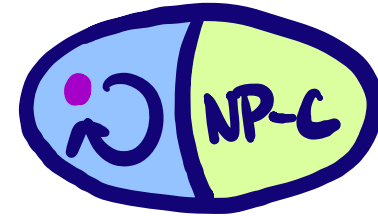
OMNI-
EXPRESSIVE

A DICHOTOMY OF EXPRESSIVITY



Thm (B. + PINSKER + SCHÖBI '25)

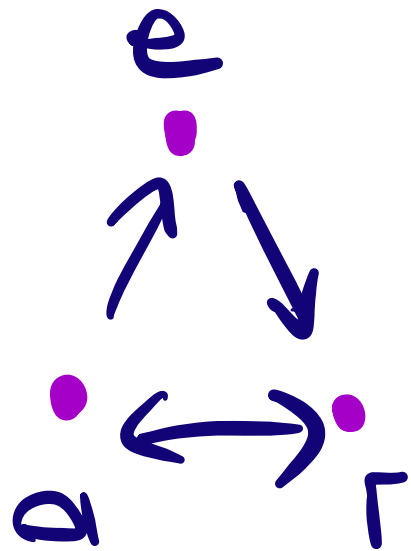
$\mathcal{A}$ TCL.



EITHER $\mathcal{A}$ IS OMNI-EXPRESSIVE
OR IT CAN ONLY EXPRESS
DIGRAPHS* WITH PSEUDO-LOOPS.

* SMOOTH, ALGEBRAIC LENGTH 1

PSEUDO-LOOPS $\Leftrightarrow$ PSEUDO-IDENTITIES



$\Leftrightarrow$

$$\cup \circ s(a, r, e, a)$$

↓ ↓ ↓ ↓

$$\vee \circ s(r, a, r, e)$$

$$\Phi = \langle (a)(r)(e)(a) \rangle_{P_{DI}(A)}$$

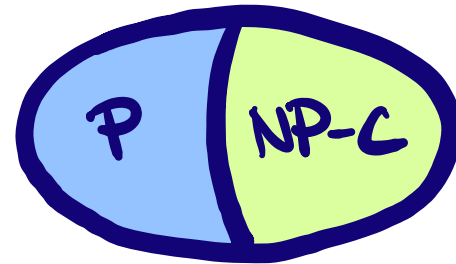
OMNI-EXPRESSIVE OR HAS PSEUDO-LOOP

A DICHOTOMY OF EXPRESSIVITY



Thm (B. + PINSKER + SCHÖBI '25)

$\mathcal{A}$ TCL.



EITHER $\mathcal{A}$ IS OMNI-EXPRESSIVE
OR $\text{Pol}(\mathcal{A}) = \text{POS}(\alpha, r, e, \alpha) \approx \text{VOS}(r, \alpha, e)$

WHY DO WE CARE?

?



QUESTION

λ A W-CAT.

IS IT TRUE THAT EITHER

λ A IS OMNI-EXPRESSIVE

OR $P_{\text{OI}}(\lambda A) = \text{PSEUDO-SIGGERS}$?

WOULD BE IMPLIED BY FULL LIFTING
OF BARTO+KOZIK+NIVEN '07.



When Janus can't express it all
He tries in vain, he hits a wall
When for I know no way he knows
His face of perhaps he shows

Funded by the EU (ERC, PROCOOP).
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