On symmetric term operations in finite Taylor algebras

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Given a finite algebra $\mathbf{A}$. For which tuples τ of variables $\mathbf{A}$ has τ-symmetric term operation?

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What algebras have $(x,y,y,z,z,z,z,\dots,z)$-symmetric operations?

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Lemma [Brady, 2022]

For every bad tuple τ there exists a finite algebra $\mathbf{A}$ with bounded width not having τ-symmetric term operations.

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TFAE for a finite idempotent algebra $\mathbf{A}$.

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- there doesn't exist an essentially unary $\mathbf{B} \in HS(\mathbf{A})$ or p-affine $\mathbf{B} \in HS(\mathbf{A})$
- for every n the algebra $\mathbf{A}$ has an n-ary term operation t that is symmetric on all good tuples

This cannot be strengthened.

Lemma [Brady, 2022]

For every bad tuple τ there exists a finite algebra $\mathbf{A}$ with bounded width not having τ-symmetric term operations.

Can we prove something similar for finite Taylor algebras?

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- singletonBLP+AIP, CLAP, and similar algorithms fail on CSP(Inv(h)).

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This is not true for $|A| > 2$ (our counter-example $Inv(\{0, 1, 2\}; h)$).

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   $\Rightarrow$ it is sufficient to prove that every symmetric affine subspace $R \subseteq \mathbb{Z}_p^n$ has a constant tuple $\Rightarrow$ p should not divide n.

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$R \subseteq \mathbf{D}^{\binom{n}{2}}$ has $\binom{n}{2}$-symmetries

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Consider a free algebra over $\{x, y\}$ and generate a relation R from

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Claim [Z, Pinsker, 2023]

Every finite Taylor algebra $\mathbf{A}$ has a k-WNU term of an arity n if there are no primes $p \leq |A|$ dividing $\binom{n}{k} \cdot k!$.

Proof:

Consider a free algebra over $\{x, y\}$ and generate a relation R from

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3. Can we characterize all algebras not having symmetric operations on $(a_1, \dots, a_n)$?

Thank you for your attention